

lattice truss, voltage distribution in a suspension insulator, critical speeds of a multicylinder engine, leading to a mechanical wave filter, and electric wave filters. (In the last, an attenuator is incorrectly called a "wave trap," which is a resonant element.)

Following the chapters appears a short section entitled "words and phrases" intended to give certain "strictly mathematical definitions" and starting with Kronecker's dictum, "God made the integers; all the rest is the work of man." It seems surprising to have this dictum endorsed by engineers and physicists: it might better be replaced by the statement, *God made both discrete and continuous physical quantities; man devised means (integers and real numbers) for representing them.* This section in general seems out of place and of little help to a reader who would be capable of using the book.

Besides a few obvious typographical errors, the reviewer noticed the following errors: last equation, p. 108, a minus sign should precede the first term in each member; the answer to Problem 12, p. 108, has the inequality reversed; the answer to Problem 16, p. 109, not only uses  $k$  for  $K$  but incorrectly gives a stable solution; the first answer to Problem 1, p. 210, incorrectly has the factor  $1/2$ ; the equation on p. 131 is written as if  $t$  were under the radical sign. (It is unfortunate that radical signs are used in place of fractional exponents throughout the book.)

This is a book which should be in the library of every engineer who is interested in the analytical development of his subject. It should be studied by mathematicians who are willing to admit that their place in society may need justification on other than purely intellectual grounds. It is well adapted as a text or for collateral study in an advanced course in applied mathematics or in theoretical mechanics. The authors are to be congratulated in so competently supplying a real need.

ALAN HAZELTINE

*Development of the Minkowski Geometry of Numbers.* By Harris Hancock. (Published with the aid of the Charles Phelps Taft Memorial Fund and of two Friends.) New York, Macmillan, 1939. 24+839 pp. \$12.00.

Professor Hancock says in the introduction of his book: "In every subject that occupies the human mind, be it history, philosophy, law, medicine, science, music, etc., there arise outstanding men who evince an innate genius in their special fields, an innateness that seems as it were of divine origin. Minkowski was one of the great mathematicians of all time." It is the aim of Hancock's book to make an

important part of Minkowski's work, his geometry of numbers, more easily accessible to the student of advanced mathematics. The author has endeavored to give an exposition of the whole of Minkowski's investigations as far as they were concerned with the geometry of numbers, following for the most part rather closely Minkowski's own publications. For the convenience of the reader, additional explanations and computations are given, numerous figures and examples are added, and in some cases missing proofs are provided.

There seems to be no need today to describe the mathematical work of Minkowski in a review. None less than Hilbert has given a picture of Minkowski in his "Gedächtnisrede" which can be found at the beginning of the *Collected Works* of Minkowski. Most mathematicians today have at least some idea about Minkowski's geometry of numbers. So we may refrain from discussing the significance of the theories developed in Hancock's book.

The table of contents is as follows: I. Preliminary notions and historical notes, II. Surfaces that are nowhere concave, III. The volume of bodies, IV. Bodies which with respect to their volumes have more than one point with integral coordinates, V. Applications of the preceding investigations, VI. Algebraic numbers, VII. Arithmetical theory of a pair of lines; theory of continued fractions and of the real quadratic irrational numbers, VIII. Shorter papers by Minkowski. Historical, IX. A criterion for algebraic numbers, X. The theory of continued fractions, XI. Periodic approximation of algebraic numbers, XII. On the approximation of a real quantity through rational numbers, XIII. A further analytic-arithmetic inequality, XIV. The arithmetic of the ellipsoid, XV. Computation of a volume through successive integrations, XVI. Proof of the new analytic-arithmetic inequality, XVII. The extreme standard bodies, XVIII. Densest placement of congruent homologous bodies, XIX. Miscellany, XX. New theory of quadratic forms. Region of discontinuity for arithmetical equivalence.

Chapter I corresponds to the first part of the third chapter of Minkowski's *Diophantische Approximationen* while the second part is treated at the beginning of Chapter XVIII. In the Chapters II to VII, Hancock follows the first four chapters of Minkowski's *Geometrie der Zahlen*; the last chapter of Minkowski's book is the subject of Hancock's Chapters XIII to XVII. The remaining parts of Hancock's book discuss the various papers of Minkowski which are concerned with the geometry of numbers, and which can be found in the *Collected Works*. In Chapter X Hancock follows the thesis of his

pupil P. Pepper who worked out proofs for theorems stated by Minkowski.

As will be seen from this description, the book restricts itself entirely to Minkowski's own work in the geometry of numbers. Professor Hancock explains in the introduction that this was done in order to limit the content of the book which otherwise would have been beyond bounds. Still, one may regret that the newer developments were not at least indicated. There is another factor which decreases the value of the book, this being that the representation often is not as good as one may wish (compare, for instance, Article 8 with the corresponding section of the *Diophantische Approximationen*). The different parts of the book could have been connected more closely. Finally, there are numerous misprints some of which are confusing. In the opinion of the reviewer, it would not be surprising, if many readers should prefer the original texts. On the other hand, there will be many mathematicians who will be very grateful to Professor Hancock for facilitating for them access to Minkowski's beautiful investigations.

RICHARD BRAUER

*Enzyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen.* Band I, Teil 1, Heft 2, 114 pp.; Band I<sub>1</sub>, Teil 1, Heft 4, 51 pp.; Band I<sub>1</sub>, Heft 5. 54+54+28 pp. Leipzig and Berlin, Teubner, 1939.

This new edition of the *Enzyklopädie der Mathematischen Wissenschaften* appears exactly forty years after the publication of the first volume of the first edition in 1899. The original project of compiling and presenting a comprehensive review of the science of mathematics and its allied fields was considered a monumental and ambitious task which aroused great interest among contemporary mathematicians. The initiative to the *Enzyklopädie* was taken by Felix Klein, Heinrich Weber and Franz Meyer and a great number of other prominent mathematicians was gradually associated with this initial group. To begin with, the work had been planned in the form of a regular encyclopedia in which the material should appear as special articles for each mathematical term. After an early attempt along these lines it became clear that this method of presentation led to considerable overlapping and the artificial classification of the subjects according to the alphabet tended to make them incoherent and lacking in general views. This prompted the fortunate decision of giving a systematic account of the field of mathematics in which the various articles on the subdivisions of the science were fitted into their natural connections as far as it was possible. It may also be remarked