curve-elements) are the group of point transformations and the union-preserving transformations from curve-elements of order n into lineal-elements, or the extensions of these two types. An additional theorem is that any transformation from curve-elements of the (x, y, z)-space into lineal-elements of the (X, Y, Z)-space, by which any union of the (X, Y, Z)-space corresponds to exactly $\infty^{2(n-1)}$ curves of the (x, y, z)-space, is union-preserving. (Received August 11, 1943).

299. L. A. MacColl: *Geometrical characterizations of some families of dynamical trajectories.*

This paper deals with a certain five-parameter family of curves, which can be regarded as the family of trajectories of an electrified particle in an arbitrary static magnetic field. A set of geometrical properties is given which completely characterizes the family of curves. Certain other related families of curves, including the four-parameter family of trajectories of the particle moving with an arbitrarily prescribed value of the energy, are also discussed and characterized by sets of geometrical properties. (Received October 1, 1943.)

300. Alice T. Schafer: *Two singularities of space curves.*

This paper uses the methods of projective differential geometry to study an analytic space curve in the neighborhood of an inflexion point and, second, a planar point. Canonical power-series expansions representing the curve in the neighborhood of each singular point are deduced by suitably choosing the projective coordinate system. These canonical expansions are then used to study properties of the curve in this neighborhood. Particular emphasis is placed on the surfaces osculating the curve, sections of the tangent developable of the curve made by the faces of the tetrahedron of reference, and projections of the curve onto the faces of the tetrahedron of reference. (Received October 1, 1943.)

Statistics and Probability

301. W. K. Feller: *On a general class of "contagious" distributions.*

This paper is concerned with some properties of a class of contagious distributions which contains, among others, some distributions studied by Greenwood and Yule, Polya, and Neyman, respectively. (Received August 3, 1943.)

302. H. B. Mann and Abraham Wald: *On the statistical treatment of linear stochastic difference equations.*

For any integer t let $x_{1t}, \cdots, x_{rt}$ be a set of r random variables which satisfy the system of linear stochastic difference equations $\sum_{j=1}^{r}\sum_{k=0}^{p_{ij}}\alpha_{ijk}x_{j,t-k}+\alpha_i=\epsilon_{it}(i=1, \cdots, r)$. The coefficients α_{ijk} and α_i are (known or unknown) constants and the vectors $\epsilon_t=(\epsilon_{1t}, \cdots, \epsilon_{rt})$ $(t=1, 2, \cdots)$ are independently distributed random vectors each having the same distribution. It is assumed that $E(\epsilon_{it})=0$. The problem dealt with in this paper is to estimate the unknown coefficients α_{ijk} and α_i on the basis of Nr observations $x_{it}(i=1, \cdots, r; t=1, \cdots, N)$. The statistics used as estimates of the unknown coefficients are identical with the maximum likelihood estimates if ϵ_t is normally distributed. The joint limiting distribution of these estimates is obtained without assuming normality of the distribution of ϵ_t. (Received August 7, 1943.)

303. Abraham Wald and Jacob Wolfowitz: *An exact test for randomness in the non-parametric case based on serial correlation.*

Let $X_1, \cdots, X_n$ be n chance variables, about the distribution of which nothing is known. Let the problem be to test the (null) hypothesis that $X_1, \cdots, X_n$ are independently distributed with the same distribution function. It is shown that an exact test of this hypothesis based on the serial correlation coefficient can be made. For this purpose the distribution of the serial correlation coefficient in the sub-population consisting of all possible permutations of the observed values is employed. Under the null hypothesis this distribution is independent of the distribution function of $X_i(i=1, \cdots, n)$. Several exact moments are obtained and asymptotic normality is proved. (Received August 7, 1943.)

Topology

304. L. M. Blumenthal: *Metric study of generalized elliptic spaces.* Preliminary report.

Let Σ be semimetric with diameter d, $\phi(x/\rho)$ a real single-valued monotonic decreasing function (ρ, positive parameter) defined over the distance set of Σ, with $\phi(0)=1$, $\phi(d/\rho)=0$. The space is called generalized elliptic $E^{\phi}_{n,\rho}$ provided: I. For each positive integer k and each $k+1$ points $p_1, p_2, \cdots, p_{k+1}$ there corresponds an *allowable matrix* (ϵ_{ij}); $\epsilon_{ii}=1$, $\epsilon_{ij}=\epsilon_{ji}=\pm 1$ $(i, j=1, 2, \cdots, k+1)$ with every nonvanishing principal minor of the determinant $|\epsilon_{ij}\phi(p_ip_j/\rho)|$ positive. II. The integer n is the smallest for which there exist $n+1$ points $p_1, p_2, \cdots, p_{n+1}$ such that $|\epsilon_{ij}\phi(p_ip_j/\rho)|$ does not vanish for any allowable matrix (ϵ_{ij}). For the ordinary elliptic space, Σ is the surface of the sphere $S_{n,\rho}$ with opposite points identified and "shorter arc" metric, while $\phi(x/\rho)\equiv\cos(x/\rho)$. An interesting feature of these spaces is that, in contrast to others (that is, euclidean, hyperbolic, spherical) the mutual distances of a set of points does not suffice to determine the dimension of the subspace which contains them. Thus a given set of three numbers may be distances of three points on an $E^{\phi}_{1,\rho}$, and also distances of three points not on any $E^{\phi}_{1,\rho}$, but on an $E_{2,\rho}$. It is found that certain pseudo-$E^{\phi}_{n,\rho}(n+3)$-tuples are contained in an $E^{\phi}_{n+2,\rho}$. New theorems concerning determinants are a by-product of the study. (Received August 3, 1943.)

305. L. M. Blumenthal: *New formulations of some imbedding theorems.*

The theorems deal with congruent imbedding of metric spaces in Hilbert space, and center about the two following results: I. A complete connected ptolemaic metric space in which every point is contained in a closed convex neighborhood is convex. II. A complete, convex, externally convex metric space in which the Theorem of Pythagoras is valid is congruently contained in Hilbert space. An application of a well known theorem of Menger-Schoenberg yields the first result when it is shown that each two points of the space are joined by an arc with everywhere vanishing metric curvature. To establish the second theorem one notes that every pair of lines that intersect at "right angles" is congruently imbeddable in the plane. It follows that the space has the *weak euclidean four-point property* and the conclusion follows from a result due to the writer. (Received August 3, 1943.)