303. Abraham Wald and Jacob Wolfowitz: *An exact test for randomness in the non-parametric case based on serial correlation.*

Let $X_1, \cdots, X_n$ be n chance variables, about the distribution of which nothing is known. Let the problem be to test the (null) hypothesis that $X_1, \cdots, X_n$ are independently distributed with the same distribution function. It is shown that an exact test of this hypothesis based on the serial correlation coefficient can be made. For this purpose the distribution of the serial correlation coefficient in the sub-population consisting of all possible permutations of the observed values is employed. Under the null hypothesis this distribution is independent of the distribution function of $X_i(i=1, \cdots, n)$. Several exact moments are obtained and asymptotic normality is proved. (Received August 7, 1943.)

Topology

304. L. M. Blumenthal: *Metric study of generalized elliptic spaces.* Preliminary report.

Let Σ be semimetric with diameter d, $\phi(x/\rho)$ a real single-valued monotonic decreasing function (ρ, positive parameter) defined over the distance set of Σ, with $\phi(0)=1$, $\phi(d/\rho)=0$. The space is called generalized elliptic $E^{\phi}_{n,\rho}$ provided: I. For each positive integer k and each $k+1$ points $p_1, p_2, \cdots, p_{k+1}$ there corresponds an *allowable matrix* (ϵ_{ij}); $\epsilon_{ii}=1$, $\epsilon_{ij}=\epsilon_{ji}=\pm 1$ $(i,j=1,2,\cdots,k+1)$ with every nonvanishing principal minor of the determinant $|\epsilon_{ij}\phi(p_ip_j/\rho)|$ positive. II. The integer n is the smallest for which there exist $n+1$ points $p_1, p_2, \cdots, p_{n+1}$ such that $|\epsilon_{ij}\phi(p_ip_j/\rho)|$ does not vanish for any allowable matrix (ϵ_{ij}). For the ordinary elliptic space, Σ is the surface of the sphere $S_{n,\rho}$ with opposite points identified and "shorter arc" metric, while $\phi(x/\rho)\equiv\cos(x/\rho)$. An interesting feature of these spaces is that, in contrast to others (that is, euclidean, hyperbolic, spherical) the mutual distances of a set of points does not suffice to determine the dimension of the subspace which contains them. Thus a given set of three numbers may be distances of three points on an $E^{\phi}_{1,\rho}$, and also distances of three points not on any $E^{\phi}_{1,\rho}$, but on an $E_{2,\rho}$. It is found that certain pseudo-$E^{\phi}_{n,\rho}(n+3)$-tuples are contained in an $E^{\phi}_{n+2,\rho}$. New theorems concerning determinants are a by-product of the study. (Received August 3, 1943.)

305. L. M. Blumenthal: *New formulations of some imbedding theorems.*

The theorems deal with congruent imbedding of metric spaces in Hilbert space, and center about the two following results: I. A complete connected ptolemaic metric space in which every point is contained in a closed convex neighborhood is convex. II. A complete, convex, externally convex metric space in which the Theorem of Pythagoras is valid is congruently contained in Hilbert space. An application of a well known theorem of Menger-Schoenberg yields the first result when it is shown that each two points of the space are joined by an arc with everywhere vanishing metric curvature. To establish the second theorem one notes that every pair of lines that intersect at "right angles" is congruently imbeddable in the plane. It follows that the space has the *weak euclidean four-point property* and the conclusion follows from a result due to the writer. (Received August 3, 1943.)

306. R. H. Fox: *The complete homotopy group.*

The fundamental group $\tau_\omega(Y)=\pi_1(Y^T)$ of the space of continuous mappings of the infinite dimensional torus T into a topological space Y has the homotopy groups $\pi_n(Y)$, $n=1, 2, \cdots$, as subgroups. The infinite symmetric group $\{P\}$ is a group of operators on τ_ω. The complete homotopy group τ is the minimal subgroup of τ_ω containing all the subgroups π_n^P. The group τ can be given a direct geometric definition and can be built up from the homotopy groups by a countable number of splitting extensions. The classical automorphisms of π_n induced by elements of π_1 are induced by inner automorphisms of τ. In fact the Abe group κ_n, $n\geqq 2$, (Jap. J. Math. vol. 16 (1940) p. 169) is the minimal subgroup containing π_1 and π_n. Similarly the multiplication $\alpha\cdot\beta\in\pi_{m+n-1}$, $\alpha\in\pi_m$, $\beta\in\pi_n$ (Whitehead, Ann. of Math. vol. 42 (1941) p. 411) is represented as a commutator $\alpha\beta^P\alpha^{-1}\beta^{-P}$ where P is a certain type of permutation. The commutators $\alpha^Q\beta^R\alpha^{-Q}\beta^{-R}$ which are not $\{P\}$-automorphs of commutators of this type are all $=1$. Generalization of the theory to groups modulo a subset B of Y is automatic. (Received October 1, 1943.)

307. D. W. Hall: *A note on primitive skew curves.*

It is shown that a locally connected continuum M not separated by the removal of any pair of its points can contain no primitive skew curve of type II unless it contains a primitive skew curve of type I. As a corollary there is the theorem of F. B. Jones (Abstract 48-11-340) to the effect that a locally connected continuum M separated by no pair of its points but by every one of its simple closed curves must be homeomorphic with a sphere provided M contains no primitive skew curve of type I. This follows immediately in view of a theorem of S. Claytor (Ann. of Math. vol. 35 (1934) pp. 809–835). (Received August 3, 1943.)

308. M. E. Shanks: *The space of metrics on a compact metrizable space.* II.

Denote by $M_0(X)$ the set of all metrics on the compact metrizable space X compatible with its topology, and by $M(X)$ the complete extension of $M_0(X)$. Then $M_0(X)$ and $M(X)$ are semi-linear normed spaces. In a previous abstract 47-5-284 the author stated that X and Y are homeomorphic if and only if $M(X)$ and $M(Y)$ are congruent. In this paper the same result is obtained from the stronger statement that the homeomorphism of X and Y implies, and is implied by, the linear isomorphism $M_0(X)$ and $M_0(Y)$. The methods are new and make essential use of the lattice of upper semi-continuous decompositions of X. (Received October 1, 1943.)

309. J. W. T. Youngs: *On surfaces of class K_1.* Preliminary report.

In a paper entitled *On the semi-continuity of double integrals in parametric form* (Trans. Amer. Math. Soc. vol. 51 (1942) pp. 336–361) Radó has proved a variety of important theorems concerning a class of surfaces designated by the symbol K_1. The purpose of this paper is to show that every surface is in the class K_1. A new functional $I^*(S)=\sup I(T, B, \|X\|)$ is discussed, the supremum being taken with respect to all representations (T, B) of the surface S for which $I(T, B, \|X\|)$ has meaning. (For the terminology see the paper of Radó.) (Received September 22, 1943.)