ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

260. R. A. Beaumont: *Groups with isomorphic proper subgroups.*

A completely reducible group is a group which is the direct product of groups of rank 1. Necessary and sufficient conditions that a completely reducible group have an isomorphic proper subgroup are found. If G is a completely reducible abelian group without elements of infinite order, then G has an isomorphic proper subgroup if, and only if, G is of infinite rank. If G is a completely reducible abelian group without elements of finite order, then G has an isomorphic proper subgroup if, and only if, either G has a direct factor which is essentially a proper subgroup of the additive group of all rational numbers or G is of infinite rank. (Received September 29, 1944.)

261. R. H. Bruck: *Quasigroup theory.* II. *The lower central series.* Preliminary report.

This abstract is a continuation of abstract 50-5-107. Let G be a loop with unit 1, $\mathfrak{G}$ be the group consisting of all products of a finite number of permutations R_x, L_y, R_z^{-1} and L_u^{-1}, where $xy=xR_y=yL_x$. R. Baer has posed the problem of determining an "inner mapping group" $\mathfrak{J}\subset\mathfrak{G}$ with the property that a subloop $H\subset G$ is normal if and only if invariant under $\mathfrak{J}$. It is shown here that $\mathfrak{J}$ is generated by the set of all permutations $R_{x,y}=R_xR_yR_{xy}^{-1}$, $M_{x,y}=R_yL_xR_{xy}^{-1}$; moreover $U\subset\mathfrak{G}$ is in $\mathfrak{J}$ if and only if $1U=1$. If H is any subloop of G and $\mathfrak{M}$ any set of mappings of G which map H into itself, the H-subloop $H(\mathfrak{M})$ generated by all elements xAL_x^{-1} with $x\subset H$, $A\subset\mathfrak{M}$ is mapped into itself by $\mathfrak{M}$. In particular if H is a normal subloop of G then $H(\mathfrak{J})$ is normal both in G and in H, and $H/H(\mathfrak{J})$ is an abelian group. The lower central series of G may be defined by $H_0=G$, $H_1=G'=H_0(\mathfrak{J})$, $H_2=H_1(\mathfrak{J})$, $\cdots$, and the notions of solubility and class are immediate. $\mathfrak{J}$ is a group of automorphisms of G only in special cases, as for example in the (non-trivial) case of a commutative Moufang loop. (Received August 21, 1944).

262. R. H. Bruck: *Quasigroup theory.* III. *Finite p-loops.* Preliminary report.

In this study a finite loop G is called a p-loop if and only if the associated group $\mathfrak{G}$ is a p-group. Also if H is a normal subloop of G then by definition $(H, G)=H(\mathfrak{J})$. (For explanation of these notations see abstract 50-11-261.) Finally a loop is said to be abelian if and only if it is a commutative group. It is then shown that many of the elementary theorems for p-groups, and even some of the proofs, remain valid when the word "group" is replaced throughout by "loop." For example, P. Hall's proof of the Burnside basis theorem survives without further change; the essential underlying theorems are valid but require new proofs. Again, the upper and lower central series exist and have equal length c, the class of the p-loop. However, as shown by construction, there exist metabelian p-loops of order p^2, for p an odd prime, in contrast with the case for groups. It is to be noted that although every p-loop has prime-power order, the converse is false; nevertheless every *p-group* is also a p-loop in the present sense. (Received September 21, 1944.)

263. A. L. Foster: *The idempotent elements of a commutative ring form a Boolean algebra; application to principal ideal rings; ring duality and general transformation theory.*

In a previous communication ([1] Foster and Bernstein, Duke Math. J. vol. 11 (1944) pp. 603–616), it was shown that the classical symmetry and duality of Boolean algebra is merely an instance of a symmetry-duality theory inherent in the general concept of ring. The present paper is mainly concerned with (a) an application of this theory to yield the extension of Stone's theorem mentioned in the title; (b) a specialization of this extension to the factor rings of principal ideal rings; (c) a remark on Boolean-like rings (a certain generalization of Boolean rings in which the ring addition has the same formal definition as in ordinary Boolean rings); (d) a preliminary location of the ring-duality theory of [1] within the framework of a more general transformation theory. (Received September 11, 1944.)

264. L. K. Hua: *On the distribution of quadratic non-residues and the Euclidean algorithm in real quadratic fields.* I.

This paper determines conditions under which the Euclidean algorithm cannot hold in a real quadratic field, by first obtaining an upper bound for the least quadratic non-residue q_1 of a given prime p. Let $p \geqq e^{250}$, and let q_1, q_2, and q_3 be the three least prime quadratic non-residues mod p. Then q_1 is at most $(60p^{1/2})^{0.625}$, while q_2 and q_3 have similar bounds with the coefficient 60 replaced by 240 and 720, respectively. These results are established by appeal to certain lemmas of Rosser (Amer. J. Math. vol. 63 (1941) pp. 211–232). Actually, similar results could be obtained under the weaker hypothesis $p \geqq e^{160}$, but the proof would be much more complicated. Using these inequalities it is proved that there is no Euclidean algorithm in the real quadratic field $R(d^{1/2})$ when d is square-free and greater than e^{250}. This extends results of Rédei and A. Brauer. (Received November 3, 1944.)

265. L. K. Hua and S. H. Min: *On the distribution of quadratic non-residues and the Euclidean algorithm in real quadratic fields.* II.

The problem of the existence of the Euclidean algorithm in quadratic fields $R(m^{1/2})$ is still unsolved only in the case that $m=p$ is a prime of form $8n+1$. In this paper it is proved that the algorithm does not exist if $p>137$ is a prime of form $24n+17$. (Received September 9, 1944.)

266. Nathan Jacobson: *A definition of the radical for an arbitrary ring.*

If A is an arbitrary ring with an identity, the radical N is defined to be the totality of elements z such that $1+za$ has a right inverse for every a in A. If A does not have an identity, the radical is defined by using the concept of a quasi-inverse due to Perlis. It is shown that N is a two-sided ideal and it coincides with the left radical, defined in a similar manner. If A has an identity, N is the intersection of the maximal right (left) ideals of A. The results of Stone and of McCoy are a consequence of this theorem. The author has also investigated the radical of an arbitrary algebra and of a normed ring and in the latter case has obtained the criterion: $z \in N$ if and only if $(za)^n \to 0$ for every a in A. (Received August 30, 1944.)

267. Jakob Levitzki: *Chain-conditions and nilpotency.*

Let S be an arbitrary ring. Denote by N_σ, N, and N_γ the sum of all nilpotent, semi-nilpotent and nil-ideals of S, respectively. R. Brauer (Bull. Amer. Math. Soc. vol. 48 (1942) pp. 752–758) by a simple argument proved that if the minimal condition holds for the ideals of S contained in N_σ, then N_σ is nilpotent. By a similar argument the author derived a characteristic condition for the nilpotency of N_σ (Duke Math. J. vol. 11 (1944) pp. 367–368). In the present note, characteristic minimal and maximal conditions are derived for the nilpotency of N_σ, N and N_γ. These results are corollaries of the following theorems: Denote by $R_1, R_2, \cdots$ resp. by $L_1, L_2, \cdots$ infinite sequences of right resp. left-ideals of a nil-subring T of S, then T is nilpotent if and only if: I. Each descending chain of the form $SL_1 \supset SL_1L_2 \supset \cdots$ and of the form $R_1S \supset R_2R_1S \supset \cdots$ is finite. II. Each ascending chain of the form $(O:L_1)_r \subset (O:L_1L_2)_r$ and of the form $(O:R_1)_l \subset (O:R_2R_1)_l \subset \cdots$ is finite. These theorems yield as a consequence various characteristic conditions for the semi-primarity of a ring. (Received September 7, 1944.)

Analysis

268. R. P. Agnew: *A simple and natural notation for the theory of summability of series and sequences.*

It is proposed that methods of summability be regarded as *operators*, and that the operational (that is, functional) notation be employed in the theory of summability. Thus the statement that a given sequence $s_0, s_1, s_2, \cdots$ or $\{s_n\}$ is summable to σ by a given method A is represented by $\sigma = A\{s_0, s_1, \cdots\}$ or $\sigma = A\{s_n\}$. The statement that a series $u_0+u_1+\cdots$ or $\sum u_n$ is summable B to σ is abbreviated to $\sigma = B\{u_0 + u_1 + \cdots\}$ or $\sigma = B\{\sum u_n\}$. Discussions and examples are given to illustrate the notation which, the author believes, should have been universally adopted many years ago. (Received September 28, 1944.)

269. R. P. Agnew: *Spans in Lebesgue and uniform spaces of translations of step functions.*

It is shown that, for each $p>1$, the closure in the Lebesgue space L_p of the linear manifold determined by the translations of a given simple step function is the whole space L_p. An explicit formula is given for the approximation of one simple step function by linear combinations of translations of another. (Received September 23, 1944.)

270. E. F. Beckenbach: *Concerning the definition of harmonic functions.*

The following result, which may be compared with the Looman-Menchoff theorem concerning the Cauchy-Riemann first order partial differential equations, is established: If the real function $u(x, y)$ and its first order partial derivatives are continuous