ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

1. Richard Brauer: *On the arithmetic in a group ring.* II. Preliminary report.

The paper is a continuation of an earlier paper (Proc. Nat. Acad. Sci. U.S.A. vol. 30 (1944) pp. 109–114). The arithmetic in the group ring of a group of finite order over suitable algebraic number fields is studied. In particular, the prime ideals are considered. In the first paper, a connection has been established between these prime ideals and the prime ideals of the group rings of certain subgroups. New relations of this kind are given. On the basis of these results, a number of statements concerning the characters of the group can be made, provided that the corresponding statements concerning the characters of the subgroups are known. If p^a is the highest power of a rational prime p dividing the order of the group, it is shown that the number of characters belonging to the same p-block lies below a bound depending only on p^a. This implies a corresponding result for the prime ideal divisors of p belonging to the same block. A refinement of the orthogonality relations for group characters is given. (Received October 16, 1945.)

2. Paul Erdös and Irving Kaplansky: *The asymptotic number of Latin rectangles.*

It has been conjectured that the number of n by k Latin rectangles is asymptotic to $(n!)^k e^{-k(k-1)/2}$. In this paper the conjecture is proved not only for k constant (as $n \to \infty$) but for $k < (\log n)^{2-\epsilon}$. Certain closer approximations are also found. (Received October 18, 1945.)

3. B. W. Jones: *Equivalence of quadratic forms over the ring of 2-adic integers.*

Any two quadratic forms f and g with coefficients in $R(p)$, the ring of p-adic integers, are equivalent in $R(p)$ to $2^{t_1}f_1+2^{t_2}f_2+\cdots+2^{t_k}f_k$ and $2^{s_1}g_1+2^{s_2}g_2+\cdots+2^{s_j}g_j$ where $t_1<t_2<\cdots<t_k$, $s_1<s_2<\cdots<s_j$ and each f_i and g_i is a form in $R(p)$ of unit determinant. If h is that portion of f containing $f_i, f_{i+1}, \cdots, f_{i+r}$ it is called an 8-block if it satisfies the following two conditions for $\lambda=8$: (1). $i=1$ or $2^{t_i}f_i \equiv 0 \pmod{4\cdot 2^{t_{i-1}}}$, (2). $i+r=k$ or $2^{t_{i+r+1}}f_{i+r+1}\equiv 0 \pmod{2^{t_{i+r}}\cdot\lambda}$. If h is not an 8-block but satisfies conditions 1 and 2 for $\lambda=4$, it is called a 4-block. These two concepts are included in the term λ-block for $\lambda=8$ or 4. The following result is proved: if f and g are two forms written as above they are equivalent in $R(2)$ if and only if $k=j$, for every i, $s_i=t_i$, and f_i and g_i

have the same number of variables and are both properly primitive or improperly primitive, and for every pair F and G of corresponding λ-blocks of f and g the following conditions hold: (a) if $\lambda=8$, then F is equivalent to G in the *field* of 2-adic numbers; (b) if $\lambda=4$, then the Hasse invariants $c_2(F)$ and $c_2(G)$ are equal and $|F|/|G|$ is a square unit or 5 times the square of a unit in $R(2)$. This criterion is simpler than previously known ones. (Received October 17, 1945.)

4. S. A. Kiss: *Transformations on lattices and structures of logic.*

Any binary operational system $(\cdot)$ is transformed into an isomorphic system $(+)$ by a one-one transformation t on its elements: $xt+yt=(x\cdot y)t$ or $(xt+yt)t^{-1}=x\cdot y$ and $x+y=(xt^{-1}\cdot yt^{-1})t$. The lattice of a Boolean algebra B^n has $n!2^n$ "link-preserving" transformations with which 2^n distinct transformed systems can be constructed from $(\cap)$. These are mutually distributive over each other and constitute, together with the "principal" transformations, a logical classification structure with 2^n truth classes in which normal forms, quantifiers and identically true formulae exist for each truth class. Consequently, there are as many different classes of logic as finite Boolean algebras B^n $(n=1, 2, 3, 4, \cdots)$ although hitherto only the 2-class logic corresponding to B^1 has been known and used. A link is an immediate connection between two elements of a discrete lattice. For example, the diagram of B^2 balanced on a vertex is a square; its 8 link-preserving (4 principal) transformations are the symmetries of the square (rectangle). These transformations define 4 distinct systems two of which are the known $(\cap)$ and $(\cup)$ and two are new. Link-preserving transformations of certain infinite distributive lattices are also studied and their connection with the number systems (integers, complex and hypercomplex numbers) is established. (Received October 12, 1945.)

5. Ivan Niven: *Sums of squares of integral quaternions.*

Every integral quaternion $a+2bi+2cj+2dij$ is expressible as a sum of three squares of integral quaternions, but not every one is a sum of two squares. (Received October 20, 1945.)

6. Gordon Pall: *On generalized quaternions.*

Let $(a_{\alpha\beta})$ denote a symmetric ternary matrix $(\alpha, \beta=1, 2, 3)$, and $(A_{\alpha\beta})$ its adjoint. Assume that the $a_{\alpha\alpha}$ and $2a_{\alpha\beta}$ are integers, and choose $\epsilon_\alpha=0$ or 1 to secure $4A_{\alpha\beta}\equiv\epsilon_\alpha\epsilon_\beta$ mod 2. Then the form $F=(t_0+2^{-1}\sum\epsilon_\alpha t_\alpha)^2+\sum A_{\alpha\beta}t_\alpha t_\beta$ is an integral quaternary quadratic form, and is the norm-form of the system of generalized quaternions associated with $(a_{\alpha\beta})$. The arithmetic of these quaternions is maximal if and only if F is fundamental, that is, F cannot be derived by an integral linear transformation from another norm-form of smaller determinant. There is a close connection between the arithmetics of these quaternions and the integral automorphs of F, and of $(a_{\alpha\beta})$, the correspondence being one-to-one in the maximal case. The factors of any given norm of a primitive quaternion are always unique, up to a unit factor. However, unless the genus of F contains only one class, such factorization is in general not possible, even when all the obviously necessary conditions are satisfied. There exist exactly 39 positive-definite classes of norm-forms in genera of one class, hence exactly 39 such systems in which factorization is in general possible as well as unique. However, only five of these are maximal. If the genus of a positive-definite norm-form F contains two different classes of norm-forms, then it also contains a class of quaternary forms which are not norm-forms. (Received November 23, 1945.)

7. Ernst Snapper: *Polynomial matrices in one variable, differential equations and module theory.*

This paper establishes the foundation for the theory of matrices $A=(\alpha_{ij})$, where $(\alpha_{ij})\in P[x_1, \cdots, x_n]$. Part I treats the case $n=1$. Contrary to the classical procedure which uses sub-determinants of A, the theory is developed intrinsically in terms of the column space C and row space R of A. The meanings of the irreducible factors and multiplicities of the norm and elementary divisor of A for C and R thus become clear. Systems of linear differential equations and algebraic equations are fully discussed. Part II reviews and extends the ideal theoretic module theory, developed by P. M. Grundy in *A generalization of additive ideal theory*, Proc. Cambridge Philos. Soc. vol. 38 (1942), and by the author in *Structure of linear sets*, Trans. Amer. Math. Soc. vol. 52 (1942). This theory is the foundation for the case $n>1$. A general theory of systems of linear equations over any ring $\mathfrak{r}$ is developed. All known criteria for the solvability of such systems for special rings are corollaries of the *criterion of lengths* of this general theory. If $\mathfrak{r}=P[x]$, the theory becomes the theory of Part I. (Received October 7, 1945.)

8. Ernst Snapper: *Polynomial matrices in several variables.*

This paper discusses the theory of matrices $A=(\alpha_{ij})$, where $\alpha_{ij}\in P[x_1, \cdots, x_n]$. The module theory, discussed in Part II of the author's paper *Polynomial matrices in one variable, differential equations and module theory*, associates several invariants to the column space C and the row space R of A, for example the associated primes $\mathfrak{p}_j$, the $\mathfrak{p}_j$-lengths, the $\mathfrak{p}_j$-elementary divisors, and so on. Since R and C are *polynomial* modules, the theory of the Hilbert characteristic function can be developed for them which gives rise to one further invariant, called the $\mathfrak{p}_j$-degree. In terms of these invariants, the theory of the system of linear partial differential equations and algebraic equations, represented by A, is investigated. Furthermore, the irreducible factors and multiplicities of the norm and elementary divisor of A, as defined by the author in *The resultant of a linear set*, Amer. J. Math. vol. 66 (1944), are explained in terms of the above invariants. (Received October 7, 1945.)

ANALYSIS

9. N. R. Amundson: *On the boundary value problem of third kind for the quasi-linear parabolic differential equation.*

The author considers the quasi-linear parabolic equation with boundary conditions of the *third* kind for the open rectangle, that is, $u_{xx}=f(x, y, u, p, q)$; $-a_1u_x+b_1u=c_1(y)$, when $x=0$; $a_2u_x+b_2u=c_2(y)$, when $x=l$; $u=\phi(x)$, when $y=0$, where $c_i(y)$ and $\phi^{(iv)}(x)$ are continuous and b_i/a_i are non-negative constants. By use of the Green's function for the problem the above system is shown to be equivalent to a nonlinear integro-differential equation. Assuming that $f(x, y, u, p, q)$ is continuous in all five variables, and that its partial derivatives with respect to y, u, p, q satisfy a Lipschitz condition in u, p, q and are bounded, the *existence* of a solution $u(x, y)$ of the integro-differential equation is proved by an iteration method. Under the further assumption the u_x and u_y satisfy a Hölder condition with respect to y, the *uniqueness* of the solution $u(x, y)$ is established. M. Gevrey (*Thèse*, Journal de mathématique (6) vol. 9 (1913) and vol. 10 (1914)) considers the same differential equation for boundary conditions of the first kind. (Received October 19, 1945.)