curves in which the developables of the congruence intersect the surface is obtained. This net is called the *intersector net* on S. It is shown that the exhibition of a congruence relative to S for which the intersector net coincides with the lines of curvature net on S requires a solution of a partial differential equation of Laplace. It is also demonstrated that the specification of a congruence for which the intersector net coincides with the asymptotic net on S requires a solution of two partial differential equations of parabolic type. (Received October 11, 1945.)

37. T. Y. Thomas: *Absolute scalar invariants and the isometric correspondence of Riemann spaces.*

Necessary and sufficient conditions for the isometric correspondence of Riemann spaces R_n and $\bar{R}_n$ are given in terms of the equality of absolute scalar invariants of the spaces. In the general case for which the spaces admit a complete set of n functionally independent scalars it is proved that these and a certain derived set of scalars suffice for the solution of the problem. The solution of the correspondence problem is given for spaces of two dimensions which do not admit two functionally independent scalars. (Received October 10, 1945.)

38. T. Y. Thomas: *Topological theory of dynamical systems.*

The projective or topological theory of dynamical systems is concerned with the study of the trajectories independently of their time parameterization. This paper deals with the possible changes in the invariants determining the system which leave the trajectories unaltered. The case of the conservative system is of especial interest and is treated in detail. Under the assumption that the dynamical systems admit essentially a single quadratic or energy integral it is proved that the most general transformation on the coefficients $g_{\alpha\beta}$ of the expression for the kinetic energy and the potential function V is given by $g_{\alpha\beta}=(cw+d)h_{\alpha\beta}$ and $V=(aw+b)/(cw+d)$ where the a, b, c, d are constants. It is shown, moreover, that the property of a conservative system of possessing essentially only one energy integral is invariant under this transformation. The methods can be applied to systems which are not conservative. (Received October 10, 1945.)

Logic and Foundations

39. A. R. Schweitzer: *On the genesis of number systems.* I.

This paper aims to effect a gradual transition from the foundations of geometry to postulates for number systems in terms of undefined relations (operations) analogous to concepts previously developed by the author, Amer. J. Math. vol. 31. The first of these sets uses the undefined operation "replacement" (transformation) analogous to that employed in Chapter II of the preceding article, p. 373. A set S of elements (α) is combined into dyads $(\alpha\beta)$ of a set T assumed subject to two types of replacement of dyads by elements, symbolized by $R(\alpha\beta)=\gamma$ and $P(\kappa\lambda)=\mu$. These relations are also expressed, $\gamma R(\alpha\beta)$ and $\mu P(\kappa\lambda)$ or $\gamma=\alpha+\beta$ and $\mu=\kappa\times\lambda$. A closer analogy is attained by assuming, instead of $R(\alpha\beta)=\gamma$, that if $\alpha\xi$, $\beta\xi(\xi\alpha, \xi\beta)$ are in T, then γ exists in S such that $\gamma\xi(\xi\gamma)$ in T replaces $\alpha\xi$, $\beta\xi(\xi\alpha, \xi\beta)$; in symbols, $R(\alpha\xi, \beta\xi)=\gamma\xi$ or $\gamma\xi R(\alpha\xi, \beta\xi)$ or $\gamma\xi=\alpha\xi+\beta\xi$; and so on. The relation $\gamma=\alpha+\beta$ then holds if and only if $\gamma\xi=\alpha\xi+\beta\xi(\xi\gamma=\xi\alpha+\xi\beta)$ for any ξ in S. Postulates in terms of $\gamma R(\alpha\beta)$ and $\mu P(\kappa\lambda)$ are also interpreted as analogous to the author's system ${}^2R_2$ (ibid. p. 382). (Received October 19, 1945.)

40. A. R. Schweitzer: *On the genesis of number systems.* II.

In continuation of the preceding paper, other developments of number systems are: (2) in terms of reflexive, symmetric and transitive relations $\alpha\beta R\gamma\delta(\alpha+\beta=\gamma+\delta)$ and $\kappa\lambda P\mu\nu(\kappa\times\lambda=\mu\times\nu)$ in analogy with the author's relation $\alpha\beta K\gamma\delta$ (ibid. p. 394) and (3) as an elaboration of linear order based on the author's system ${}^1R_1$ (ibid. p. 378). In system (2) the author employs the binary relational symbol $\alpha\beta RP\gamma\delta$ to express $R\alpha\beta=P\gamma\delta$ or $\alpha+\beta=\gamma\times\delta$; $\alpha\beta RR\gamma\delta$ and $\kappa\lambda PP\mu\nu$ are assumed respectively equivalent to $\alpha\beta R\gamma\delta$ and $\kappa\lambda P\mu\nu$. Further assumptions are: 1. $\alpha\beta RP\gamma\delta$ implies $\gamma\delta PR\alpha\beta$. 2. $\alpha\beta XY\lambda\mu$, $\lambda\mu YZ\gamma\delta$ imply $\alpha\beta XZ\gamma\delta$, where X, Y, Z are on the set (R, P). 3. There exist uniquely in S the distinct elements ω (zero) and ϵ (unity) such that $\alpha\omega RP\alpha\epsilon$ for any α in S. 4. $\alpha\omega RP\beta\epsilon$ implies $\alpha=\beta$. 5. For any α, β in S there exist uniquely γ, δ such that $\alpha\beta R\gamma\omega$ and $\alpha\beta P\delta\epsilon$. 6. $\alpha\beta R\xi\omega$ and $\beta\gamma R\eta\omega$ imply $\alpha\eta R\xi\gamma$; and similarly for the relation P. Correspondingly, distributive, inversive and commutative properties are stated. The extension of the preceding system S in analogy with the author's systems ${}^nR_n$ and ${}^nK_n$ $(n=1, 2, 3, \cdots)$ is discussed. (Received October 19, 1945.)

Statistics and Probability

41. Will Feller: *Note on the law of large numbers and "fair" games.*

An example is exhibited to show that a game can be "fair" in the sense that the expectation of loss vanishes, and nevertheless such that the probability tends to one that after n trials there will be a positive loss L_n; the ratio of L_n to the accumulated entrance fees tends to zero as slowly as one pleases. On the other hand, in the classical Petersburg game entrance fees can be determined so that the game becomes fair in the sense that the probable loss or gain will be of smaller order of magnitude than the accumulated entrance fees. (To appear in the Annals of Mathematical Statistics.) (Received October 4, 1945.)

42. Will Feller: *On the normal approximation to the binomial.*

The goodness of the normal approximation to $T_{\lambda,\nu}=\sum_{k=\lambda}^{\nu} C_{n,k}p^k(1-p)^{n-k}$ is studied with particular reference to the practically important cases of small $np(1-p)$ and of comparatively large λ and ν. Limits of the integral are determined which depend quadratically on λ and ν and are such that the integral will approximate $T_{\lambda,\nu}$ from above or from below. The relative error is also studied. In a sense this paper continues a well known series of studies by Serge Bernstein (the latest in Izvestia Akademiia Nauk SSR, 1943). By the departure from the classical, but arbitrary, use of moments unexpected simplifications are obtained which render S. Bernstein's results more accurate and valid under less stringent conditions. (To appear in the Annals of Mathematical Statistics.) (Received October 4, 1945.)

43. P. R. Halmos: *The theory of unbiased estimation.*

Let $F(P)$ be a real-valued function defined on a subset E of the set D of all probability distributions on the real line. A function f of n real variables is an unbiased estimate of F if for every system $X_1, \cdots, X_n$ of independent random variables with the common distribution P the expectation of $f(X_1, \cdots, X_n)$ exists and equals $F(P)$, for all P in E. Under the assumption that E contains all purely discontinuous distributions, the class of all functions $F(P)$ which possess an unbiased estimate is characterized and all unbiased estimates of each such F are described. It is shown that there is in every case a unique symmetric unbiased estimate which coincides also