of the lattice. Conversely, the lattice of subspaces of every projective space is complete, atomic, complemented, and modular. Every irreducible projective space not a projective plane is a coordinate space, and is determined by a division ring and by a cardinal number which may be infinite, namely its dimension. In particular, points, lines, planes and hyperplanes may be added to continuous geometries by the use of lattice ideals. This may permit the introduction of coordinates into continuous geometries by the method of von Staudt. (Received May 11, 1946.)

226. Irving Kaplansky: *Topological rings.*

The paper is devoted to the study of (i) the radical (in the sense of Jacobson, Amer. J. Math. vol. 67 (1945)) of a topological ring, (ii) the structure of compact rings, (iii) rings of functions, (iv) rings of endomorphisms. The radical is closed if the quasi-regular elements are either open or closed, but an example is given of a ring whose radical is not closed. The radical of a compact ring is the union of all (topological) nil ideals. A compact semisimple ring is continuously isomorphic to a complete direct sum of finite simple rings. Compact integral domains are local rings—possibly without chain condition. The ring of functions from a topological space S to a topological ring A is studied; results known when A is the reals or complexes are proved for any simple ring A provided S is totally disconnected. Ideal theory in the ring is also examined. The regular representations of a topological ring and the structure of the ring of continuous endomorphisms of certain topological groups comprise the final section. (Received May 31, 1946.)

227. J. C. C. McKinsey: *On the representation problem for projective algebras.*

C. J. Everett and S. Ulam (*Projective algebra* I, Amer. J. Math. vol. 68 (1946) pp. 77–88) have recently given postulates for projective algebras, and have solved the representation problem for all complete atomic projective algebras. The author shows that every projective algebra is isomorphic to a subalgebra of a complete atomic projective algebra; this result, combined with that of Everett and Ulam, provides a solution of the representation problem for all projective algebras. The method of proof is similar to the method used by M. H. Stone to establish a representation theorem for Boolean algebras. If A is an additive prime ideal, then A_x is defined to be the additive prime ideal which contains all elements of the form a_x, for a in A. If A and B are additive prime ideals, then $A\square B$ is defined to be the class of all additive prime ideals C which contain all elements of the form $a\square b$, where a is in A and b is in B. These definitions are extended in an obvious way so as to permit operations on classes of prime ideals. (Received April 29, 1946.)

ANALYSIS

228. Richard Bellman: *Stability of difference equations.*

Using methods developed in the study of stability of differential equations (cf. the author's paper, *The stability of solutions of linear differential equations*, Duke Math. J. vol. 10 (1943)), the matrix difference equation $X_{n+1}=(A+B)X_n$, where B is a perturbation matrix, is treated, and various hypotheses are made as to the solutions of $X_{n+1}=AX_n$ and the nature of B. Analogues of results of Liapounoff for differential equations are also obtained. (Received April 10, 1946).

229. Stefan Bergman: *On functions which satisfy certain systems of partial differential equations.*

Using methods of previous papers, the author studies flows in the pseudo-logarithmic (θ, λ)-plane. He introduces flows $\mathcal{J}^{(L,k)}(\phi^{(L,k)}, \psi^{(L,k)})$, $k=1, 2$, in which ϕ is the potential function and $\phi^{(L,2)}$ a fundamental solution of $\Delta\phi - N(\partial\phi/\partial\lambda)=0$, ψ is the stream function and $\psi^{(L,1)}$ a fundamental solution with singularity at (θ_0, λ_0) of $\Delta\psi + N(\partial\psi/\partial\lambda)=0$. The flows $\mathcal{J}^{(n,k)}(\phi^{(n,k)}, \psi^{(n,k)})$, $n=1, 2, 3, \cdots$; $k=1, 2$, $\psi^{(n,1)} = (\partial^n\psi^{(L,1)}/\partial\theta^n)$, $\phi^{(n,2)} = (\partial^n\phi^{(L,2)}/\partial\theta^n)$, are infinite of the nth order at (θ_0, λ_0), have *both* components single-valued in the schlicht (θ, λ)-plane. By $x=x(\theta, \lambda)$, $y=y(\theta, \lambda)$ transition to the physical plane is defined. The author computes for the flows $\mathcal{J}^{(L,k)}$, $\mathcal{J}^{(n,k)}$ pairs of numbers, $(X^{(L,k)}, Y^{(L,k)})$, $(X^{(n,k)}, Y^{(n,k)})$ respectively, called T-periods which depend only upon θ_0 and λ_0. Let $\mathcal{J}$ be a flow whose stream function is $\alpha\psi^{(L,1)} + \beta\psi^{(1,1)} + \gamma\psi^{(1,2)} + \psi_1$, where ψ_1 is a regular function. The necessary and sufficient conditions that $\mathcal{J}$ represent in the physical plane a flow around a closed curve are $\alpha X^{(L,1)} + \beta X^{(1,1)} + \gamma X^{(1,2)} = 0$ and $\alpha Y^{(L,1)} + \beta Y^{(1,1)} + \gamma Y^{(1,2)} = 0$. (Received May 31, 1946.)

230. R. P. Boas: *The rate of growth of an analytic function determined from its growth on a sequence of points.*

Theorems of V. Bernstein and of Levinson (*Gap and density theorems*, New York, 1940, chap. 7) are given simplified proofs and still further sharpened. The proofs depend on the following lemma of Phragmén-Lindelöf type: let $\phi(z)$ be analytic in $x \geqq 0$, $|\phi(z)| \leqq e^{c|z|}$, and $|\phi(re^{i\theta})| \leqq A \exp\{r\cos\theta - r\delta(r)\} + B$ for $\pi/4 \leqq |\theta| \leqq \pi/2$, where $\delta(r) \geqq 0$, $\delta(r) \to 0$, and $\int_1^\infty r^{-1}\delta(r)dr = \infty$. Then $\phi(z)$ is bounded in $x \geqq 0$. (Received April 2, 1946.)

231. W. F. Eberlein: *Weak sequential compactness and regularity of Banach spaces.*

Although it is well known that the unit sphere of a reflexive Banach space is weakly sequentially compact, the converse theorem has been obtained only under such additional restrictions as separability (Banach), strict convexity (Milman), or compactness in the sense of directed systems (Goldstine and others). It is shown that these restrictions can be dropped: A Banach space is reflexive if and only if its unit sphere is weakly sequentially compact. The proof involves a theorem of Mazur and properties of the weak neighborhood topologies. (Received May 31, 1946.)

232. Evelyn Frank: *The real parts of the zeros of a complex polynomial.*

Let $P(z)$ be a polynomial of degree n with complex coefficients and $P^*(z) = \overline{P}(-z)$, where $\overline{P}(z)$ is obtained by replacing the coefficients of $P(z)$ by their complex conjugates. If ξ is a complex constant such that $|P^*(\xi)| > |P(\xi)|$, then the polynomial of order $n-1$, $P_1(z) = [P^*(\xi)P(z) - P(\xi)P^*(z)]/(z-\xi)$, has one zero less than $P(z)$ with real part of the same sign as $\mathrm{Re}(\xi)$ and the same number of zeros as $P(z)$ with real parts of opposite sign to $\mathrm{Re}(\xi)$. By repetition of this reduction theorem, after $n-1$ steps the number of zeros of $P(z)$ with positive real parts and the number with negative real parts are found. By the use of this method and successive approximations, the real and imaginary parts of the zeros of $P(z)$ may actually be computed. (Received May 31, 1946.)

233. M. R. Hestenes: *An alternate sufficiency proof for the normal problem of Bolza.*

The purpose of the present paper is to show that a sufficiency theorem for the problem of Bolza can be obtained from the sufficiency theorem for the simpler case in which there are no differential side conditions. More precisely it is shown that if an arc C_0 is a normal arc satisfying the usual sufficient conditions for the problem of Bolza, one can add to the integrand of the function $I(C)$ to be minimized a linear combination of the functions defining the differential side conditions so as to obtain a function $J(C)$ such that C_0 satisfies the usual sufficient conditions for the problem of minimized $J(C)$ without restricting the arcs to satisfy the differential side conditions. The arc C_0 will afford a relative minimum to $J(C)$ in the class of all neighboring arcs satisfying the end conditions and therefore will minimize $I(C)$ in case the arcs are restricted to satisfy the differential equations. (Received April 17, 1946.)

234. William Karush: *A semi-strong minimum for a double integral problem in the calculus of variations.*

The author considers the problem of minimizing an integral $I(S)=\iint_A f(x, y, z, z_x, z_y)dxdy$ in a class of surfaces S: $z=z(x, y)$ defined by functions which are Lipschitzian on A and its boundary, and which have common boundary values. Define $D^2(z)=\iint_A(z^2+z_x^2+z_y^2)dxdy$. The following theorem is proved. Let S_0: $z=z_0(x, y)$ be a surface of class C'' satisfying the conditions: (i) S_0 is an extremal surface, (ii) f E-dominates $L=(1+z_x^2+z_y^2)^{1/2}$ near S_0 (cf. Hestenes, Trans. Amer. Math. Soc. vol. 60), (iii) there exists $\theta>0$ such that $I_2(\zeta)\geqq\theta D^2(\zeta)$, where I_2 is the second variation of I along S_0, and ζ is an arbitrary variation vanishing on C. Then for every finite $M>0$ there exists $\epsilon>0$ and a neighborhood U of S_0 in (x, y, z)-space such that $I(S)-I(S_0)>\min[\epsilon, \epsilon D^2(z-z_0)]$ for every S in U coinciding with S_0 on C and having $|z_x|+|z_y|<M$ almost everywhere on A. It is shown that E-dominance is equivalent to nonsingularity and the Weierstrass condition II_N. For special integrands a strong relative minimum is obtained (cf. F. G. Myers, Duke Math. J. (1943)). The proof is based on a convergence theorem analogous to one of Morrey (Duke Math. J. (1940)). (Received May 29, 1946.)

235. G. W. Mackey: *On convex topological linear spaces.*

In this paper the theory of linear systems developed in an earlier article (Trans. Amer. Math. Soc. vol. 57 (1945) pp. 155–206) is applied to the study of convex topological linear spaces. A linear system is associated with every convex topological linear space and the properties of the latter correlated with those of the linear system and with the strength of its topology relative to the strengths of the other convex topologies associated with the same linear system. A more detailed description of the contents of this paper will be found in Proc. Nat. Acad. Sci. U.S.A. vol. 29 (1943) pp. 315–319. (Received April 4, 1946.)

236. G. R. MacLane: *Concerning the uniformization of certain Riemann surfaces allied to the inverse-cosine and inverse-gamma surfaces.*

Any Riemann surface F_w obtained from the inverse-cosine surface by displacing the branch-points along the real axis is parabolic. The corresponding entire function $w=f(z)$ is such that $f'(z)=e^{-az^2+bz}\prod_{-\infty}^{\infty}\{(1-z/c_n)e^{z/c_n}\}$, with $a\geqq 0$, b real, c_n real, $c_n\to\pm\infty$ as $n\to\pm\infty$. Conversely the image of the z-plane by any entire function of this form is a distorted inverse-cosine surface. Analogous results are obtained for the surfaces "like" those defined by $w=\cos z^{1/2}$ and $w=1/\Gamma(z)$. These results follow from a study of the polynomial surfaces analogous to the surface in question. (Received May 23, 1946.)

237. Harry Pollard: *The mean convergence of orthogonal series of polynomials.*

The literature concerning series of orthogonal polynomials is devoted chiefly to the questions of ordinary convergence or summability. In the present paper the problem of mean convergence is discussed for the classical polynomials. For example it is established that if $f(x)\in L^p$, its Legendre series converges to it in pth mean if $4/3<p<4$, but not necessarily otherwise. A more complete abstract appears in Proc. Nat. Acad. Sci. U.S.A. vol. 32 (1946) pp. 8–10. (Received April 8, 1946.)

238. M. H. Protter: *Generalized spherical harmonics.*

A class of solutions, $\{\overline{R}_{n,\nu}(x, y, z)\}$, of the equation $Lu=\{\sigma_1(x)\tau_1(y)\pi_1(z)u_x\}_x+\{\sigma_2(x)\tau_2(y)\pi_2(z)u_y\}_y+\{\sigma_2(x)\tau_1(y)\pi_3(z)u_z\}_z=0$ is considered which possesses the following properties: (1) For each n and ν, $\overline{R}_{n,\nu}(x, y, z)$ is a solution of $Lu=0$ regular in the entire plane. (2) The functions $\overline{R}_{n,\nu}(x, y, z)$ can be computed by quadratures. (3) Every solution of $Lu=0$ can be expanded uniquely in an absolutely convergent series in terms of the functions $\overline{R}_{n,\nu}(x, y, z)$ where the coefficients of the expansion are given by simple differential formulas. In the special case that $Lu=0$ reduces to the Laplace equation the functions $R_{n,\nu}(x, y, z)$ become homogeneous, harmonic polynomials of degree n. Relations are found between these polynomials, Bessel functions, and associated Legendre functions. (Received May 10, 1946.)

239. I. E. Segal: *Irreducible representations of operator algebras.*

It is shown that a uniformly closed self-adjoint algebra of bounded operators on a Hilbert space has a complete set of irreducible, adjoint-preserving, continuous representations by bounded operators on (not necessarily separable) Hilbert spaces, irreducibility signifying that there exists no nontrivial closed invariant subspace. In the case of a certain operator algebra on the space of complex-valued functions square-integrable relative to Haar measure over a locally compact group, such a representation induces in a natural way an irreducible, strongly continuous, unitary representation of the group. A recent theorem of Gelfand and Rykov (Rec. Math. (Mat. Sbornik) N.S. vol. 13 (1943) pp. 301–316) is a corollary. The proofs exploit devices previously employed by Gelfand and Neumark (ibid. vol. 12 (1943) pp. 197–213), Gelfand and Rykov, and the author. (Received April 25, 1946.)

240. Otto Szász: *On the Möbius inversion formula and closed sets of functions.*

The author gives a generalization of the Möbius inversion formula and conditions for its validity. Application is made to establish completeness of certain sequences of functions of the type $f(nt)$, $n=1, 2, 3, \cdots$, where $t\geqq 0$, and $f(t)\in L_r$, or $f(t)$ is continuous in some interval. Some particular results are: (1) $f(t)=(\sin t)^\alpha$ for $0\leqq t\leqq \pi$, $f(-t)=-f(t)=f(2\pi-t)$. In this case the span of the sequence $\{f(nt)\}$ is identical with the span of the sequence $\{\sin nt\}$ in $C(0, \pi)$ for $\alpha>1/3$; for $\alpha=1/3$ the sequence $\{f(nt)\}$ is complete in $L_\infty(0, \pi)$. (2) If $f(t)=\sin t+\sum_2^\infty a_n \sin nt$ and if $\sum_2^\infty |a_n|\leqq 1$, then the sequence $f(nt)$ is complete in $L_\infty(0, \pi)$. (3) Let $f(t)=t-[t]-1/2, f(n)=0$; the sequences $\{f(nt)\}$ and $\{\sin 2n\pi t\}$ have the same span in $C(0, 1/2)$. (4) $f(t)=\operatorname{sgn}\sin n\pi t$; the sequence $\{f(nt)\}$ is complete in $L_r(0, 1)$ for all $r>1$. (5) Each of the sequences $1, e^{-t}, t/(e^{nt}-1)$; $1, e^{-t}, t^2/(e^{nt}-1)^2$, $n=1, 2, 3, \cdots$, is complete in $C(0, \infty)$. (Received May 31, 1946.)

241. Hing Tong: *Ideals of normed rings associated with topological spaces*. Preliminary report.

Let R be a perfectly normal bicompact space, $\mathfrak{R}$ the normed abelian ring of complex-valued functions continuous over R (for $f\in\mathfrak{R}$, $\|f\|=\max_R|f(x)|$), $\mathfrak{J}$ any (closed) ideal in $\mathfrak{R}$. Then $\mathfrak{J}$ is a principal ideal. If $\prod_{f\in\mathfrak{J}}\mathfrak{N}_f=N_{\mathfrak{J}}$ ($\mathfrak{N}_f$ denotes the set of zeros of f) corresponds to $\mathfrak{J}$, the correspondence is an isomorphism (that is, $\mathfrak{J}\leftrightarrow N_{\mathfrak{J}}$) such that ($\alpha_1$) $\mathfrak{J}_1\cup\mathfrak{J}_2\leftrightarrow N_{\mathfrak{J}_1}\cdot N_{\mathfrak{J}_2}$, ($\alpha_2$) $\mathfrak{J}_1\cap\mathfrak{J}_2\leftrightarrow N_{\mathfrak{J}_1}+N_{\mathfrak{J}_2}$, ($\alpha_3$) $\mathfrak{R}\leftrightarrow\{0\}$, ($\alpha_4$) $\sum_\alpha\mathfrak{J}_\alpha\leftrightarrow\prod_\alpha N_{\mathfrak{J}_\alpha}$, ($\alpha_5$) $\prod_\alpha\mathfrak{J}_\alpha\leftrightarrow$closure of $\sum_\alpha N_{\mathfrak{J}_\alpha}$. Conversely, let $\mathfrak{R}$ be the normed ring of bounded complex-valued functions continuous over a topological space R. The conditions $\mathfrak{J}\leftrightarrow N_{\mathfrak{J}}$, ($\alpha_1$) holds, and every $\mathfrak{J}\subseteq\mathfrak{R}$ is principal imply that R is a perfectly normal bicompact space. If R is a T_1' (a space $\ni y\in\bar{x}\rightarrow\bar{x}=\bar{y}$) bicompact normal space, the above results hold providing "principal ideal" is replaced by "ideal." The results also hold for bicompact normal spaces if isomorphism is relaxed to homomorphism. The ring of continuous mappings (continuous in the strong topology) of R into the ring of bounded linear operators over a Banach space with a basis leads to the same results if an ideal means a two-sided ideal. The results do not hold for non-separable Banach spaces. (Received April 10, 1946.)

Applied Mathematics

242. Garrett Birkhoff: *Symmetric Lagrangian systems*.

Let Ω be any Lagrangian dynamical system with no potential energy and kinetic energy function $L=2^{-1}E_{jk}\dot{q}_j\dot{q}_k$. Suppose that L is invariant under a simply transitive group G of rigid motions on its configuration space. Then the "generalized force" required to maintain motion along a one-parameter subgroup in the jth coordinate direction has the components $Q_i=c^{ij}_{\ k}E_{jk}$, where the $c^{ij}_{\ k}$ are the structure constants of G. It is a corollary that the d'Alembert paradox would take in non-Euclidean geometry the following form. A rigid body moving under translation, rotation, or screw motion along an axis in an incompressible, nonviscous fluid without circulation will experience no thrust along or torque about the axis. However cross-force is possible. (Received May 13, 1946.)

243. R. J. Duffin: *Nonlinear networks*. III.

A system of n nonlinear differential equations is shown to have a periodic solution. The interest of these equations is that they describe the vibrations of electrical networks under a periodic impressed force. Consider an arbitrary linear network of inductors, resistors and capacitors which does have a solution for a given periodic impressed force. The main result of this note states that the existence of a periodic solution is still guaranteed if the linear resistors of such a network are replaced by *quasi-linear* resistors. A quasi-linear resistor is one whose differential resistance lies between positive limits. No other sort of nonlinearity besides this type of nonlinear damping is considered. The proof rests on the closure properties of nonlinear transformations in Hilbert space. (Received May 7, 1946.)