$w=(N-1)^{1/2}s/\chi_{1-\alpha}$. It is shown that if D be defined as follows: (1) if $L_1 \leqq \bar{x} \leqq L_2$, $D=(2\pi)^{-1/2}\int_{(L_1-\bar{x})/w}^{(L_2-\bar{x})/w} \exp\{-y^2/2\}dy$, (2) $D=G$ otherwise, then $|P\{D \leqq \gamma\}-\alpha|$ approaches zero as $N\rightarrow\infty$. Thus D is a large sample lower confidence limit. The extension to upper and two-sided limits presents no difficulty. (Received July 8, 1946.)

Topology

336. R. F. Arens: *Convex topological algebras.* Preliminary report.

A convex topological algebra A is a convex topological linear space in which a multiplication of elements is defined, which is, as is addition and scalar multiplication (for definiteness, take the case of real scalars), continuous simultaneously in both factors. This is a generalization of the concept of normed rings. However, the elements with inverses do not form an open set, nor is inversion continuous when possible. The author proves that if A is a division algebra, and is complete in some metric, then A is finite-dimensional, and hence its structure follows from Frobenius' theorem. This result is fundamental for the representation theory of convex topological algebras. (Received June 7, 1946.)

337. R. F. Arens: *Duality in topological linear spaces.*

Let L be any topological linear space with elements x. Let L^* be the set of continuous linear functionals f defined on L, and use in L^* the topology in which convergence of directed sets means uniform convergence on each compact subset of L (the k-topology). Let this construction be repeated, using L^* instead of L, and giving rise to L^{**}, with elements X. Then for each X there is an $x\in L$ such that $X(f)=f(x)$ for each $f\in L^*$. This natural mapping from L^{**} back into L is 1-1 and continuous if L is convex; if furthermore L is complete in some invariant metric (in particular, if L is a Banach space) then the natural mapping is bicontinuous. (Received July 10, 1946.)

338. R. H. Bing: *Skew sets.*

No plane set G contains a collection of five mutually separated sets such that the closure of the sum of any pair of these five sets is the closure of a connected subset of G which is open in G. (Received July 10, 1946.)

339. G. D. Birkhoff and D. C. Lewis: *Chromatic polynomials.*

The number of ways a map P_{n+3} of $n+3$ regions can be colored with λ colors is given by a polynomial $P_{n+3}(\lambda)$ of degree $n+3$. Certain new properties of these chromatic polynomials are established. For instance, if P_{n+3} is regular and if $(-1)^h a_h$ is the coefficient of $(\lambda-2)^{n-h}$ in the expansion of $P_{n+3}(\lambda)/\lambda(\lambda-1)(\lambda-2)$ in powers of $\lambda-2$, it is shown that binomial coefficient $C_h^n \leqq a_h \leqq C_h^{n+h-1}$. Similar results are obtained for expansions in powers of $\lambda-5$. Moreover, extensive numerical calculations indicate that both $P_{n+3}(\lambda)/\lambda(\lambda-1)(\lambda-2)-(\lambda-3)^n$ and $(\lambda-2)^n-P_{n+3}(\lambda)/\lambda(\lambda-1)(\lambda-2)$ are positively completely monotonic for $\lambda\geqq 4$. This conjecture is a very strong form of the usual four-color proposition that $P_n(4)>0$. In connection with reducibility, reduction formulas, and the analysis of rings, the theory of Kempe chains, which has been applied qualitatively with considerable success to the case $\lambda=4$, is generalized so as to yield quantitative results on chromatic polynomials for all values of λ. Typical results on reducible configurations, previously obtained only by use of Kempe chains, are also obtained inductively. The present paper therefore to some extent attempts to bridge the gap between two previously separated methods of attacking the four-color problem. (Received June 27, 1946.)

340. Herbert Federer: *Dimension and measure.*

According to Szpilrajn the dimension of a separable metric space X does not exceed p, $\dim X \leqq p$, if and only if X has a homeomorph $Y \subset E_{2p+1}$ such that the $(p+1)$-dimensional Hausdorff measure of Y equals zero, $H^{p+1}_{2p+1}(Y)=0$. (See Hurewicz and Wallman, *Dimension theory*, chap. 7.) In the present paper it is shown that the foregoing statement remains true if the Hausdorff measure H^{p+1}_{2p+1} is replaced by the $(p+1)$-dimensional integralgeometric Favard measure F^{p+1}_{2p+1}. (For its definition see Federer, *The* (ϕ, k) *rectifiable subsets of* n *space*, Bull. Amer. Math. Soc. Abstract 52-5-145.) Now suppose $A \subset E_n$, $H^k_n(A) < \infty$ and B is the set of all those points of A at which A does *not* have a k-dimensional approximate tangent plane. Then Szpilrajn's theorem implies that $\dim B \leqq \dim A \leqq k$. However it was proved by the writer (in the paper quoted above) that $F^k_n(B)=0$. Hence the new theorem implies that $\dim B \leqq k-1$. This inequality is the best possible. (Received July 10, 1946.)

341. Witold Hurewicz: *Algebraic and topological classification of mappings.*

Let X and Y be locally finite polytopes of finite dimensions. Given an abelian group G, denote by $H^m(X, G)$, $H^m(Y, G)$ the m-dimensional cohomology groups of X and Y, with coefficients from the group G. If f_1 and f_2 are continuous mappings of X into Y, *f_1 is homologous to f_2 in dimension m*, if for any abelian group G the mappings f_1 and f_2 induce identical homomorphisms of $H^m(Y, G)$ into $H^m(X, G)$; and *f_1 is homotopic to f_2 in dimension m*, if f_1 and f_2 are homotopic when restricted to the m-dimensional skeleton of X. For $m \geqq 2$, π_m is the m-dimensional homotopy group of Y, and H_m the m-dimensional homology group of Y with integer coefficients. The natural homomorphism of π_m into H_m induces a homomorphism of $H^m(X, \pi_m)$ into $H^m(X, H_m)$. The following assumptions are made: (1) Y is simple in dimension m (this means that a continuous image of the m-sphere in Y determines uniquely an element of π_m). (2) The natural homomorphism of $H^m(X, \pi_m)$ into $H^m(X, H_m)$ is an isomorphism (between $H^m(X, \pi_m)$ and a subgroup of $H^m(X, H_m)$). Under these conditions the theorem holds: *If f_1 is homotopic to f_2 in dimension $m-1$ and homologous to f_2 in dimension m, then f_1 is homotopic to f_2 in dimension m.* (Received July 19, 1946.)

342. Fred Supnick: *A theorem on rectilinear deformation.*

Let G_i, $i=1, \cdots, n$, be simple closed polygons with G_i in the interior of G_{i-1}. Let rectilinear suspended chains lying in the interior of the ring bounded by G_{i-1} and G_i join a vertex of G_{i-1} to a vertex of G_i such that no two chains have the same end points or intersect each other. The author proves that any such graph can be rectilinearly deformed so that the G_i become convex and the chains rectified. (Received July 13, 1946.)

343. Fred Supnick: *Topology of sphere clusters.* I.

The author defines a *regular n-sphere cluster* as a connected set of mutually external, equal Euclidean n-spheres, each tangent to exactly N others. N is called the *degree* of the cluster. The regular circle clusters fall into classes of degrees 0, 1, 2, and 3.

The structural properties of these classes are studied. The regular 3-sphere clusters fall into classes of degrees 0, 1, · · · , 8. Certain sets of regular 3-sphere clusters of degrees 2, 3, 4, and 5 are constructed and their structural properties studied. The author has constructed an infinite set of regular 3-sphere clusters of degree 6. These have a complicated structure. Sphere clusters of higher dimension are also considered. (Received July 13, 1946.)

344. Fred Supnick: *Topology of sphere clusters.* II. *Analogue of Kuratowski's theorem.*

Let a sphere cluster of equal, mutually external, Euclidean 3-spheres be given. The linear graph whose vertices are the centers of the spheres and whose edges are the line segments joining the centers of two spheres if and only if they are tangent is called by the author the *structural graph* of the cluster. The author calls a sphere cluster planar if the structural graph of the cluster is planar. It is proved that (1) every sphere cluster of less than eight spheres must be planar and (2) eight spheres are sufficient to obtain each of the non-planar graphs appearing in Kuratowski's theorem. (Received July 13, 1946.)

345. Sister Petronia Van Straten: *Toroidal and non-toroidal graphs.*

A one-dimensional complex of 10 points 0, · · · , 9, and 30 arcs 01, 02, 03, 05, 06, 09, 12, 13, 16, 17, 18, 23, 24, 25, 28, 34, 37, 39, 45, 47, 48, 49, 56, 58, 59, 67, 68, 69, 78, 79 which are mutually disjoint except for terminal points is called a Desargues graph. The Desargues graph is irreducibly non-toroidal in the sense that it is not homeomorphic with a subset of the torus while the sum of every 29 of the 30 segments can be embedded into the torus. The Desargues graph can be embedded into the sphere with two handles. The complex of 9 points 1, · · · , 9 and 27 disjoint arcs 12, 23, 31, 45, 56, 64, 78, 89, 97, 14, 47, 71, 25, 58, 82, 36, 69, 93, 35, 57, 73, 24, 49, 92, 68, 81, 16 is called a Pappus graph. The Pappus graph can be topologically embedded into the torus. The names of the graphs are derived from the corresponding projective configurations. (Received June 28, 1946.)

346. J. W. T. Youngs: *The representation problem for Fréchet surfaces.* Preliminary report.

This paper contains a solution of the representation problem for Fréchet surfaces in which the base space is the closure of a region on the place bounded by a finite number of Jordan curves. The solution is a generalization of a result announced in Bull. Amer. Math. Soc. Abstract 52-5-221. (Received July 5, 1946.)