THE APRIL MEETING IN CHICAGO

The four hundred fifty-seventh meeting of the American Mathematical Society was held at the University of Chicago, Chicago, Illinois, on Friday and Saturday, April 28–29, in conjunction with meetings of the Institute of Mathematical Statistics. Approximately 500 persons attended, including the following 208 members of the Society:

A. A. Albert, W. R. Allen, D. N. Arden, K. J. Arnold, Max Astrachan, W. L. Ayres, Reinhold Baer, R. G. Bartle, A. F. Bausch, J. H. Bell, L. D. Berkovitz, S. F. Bibb, K. E. Bisshopp, L. M. Blumenthal, W. M. Boothby, D. G. Bourgin, H. R. Brahana, Richard Brauer, C. F. Briggs, Ellen F. Buck, R. C. Buck, E. L. Buell, I. W. Burr, B. G. Carlson, R. E. Carr, E. D. Cashwell, T. E. Caywood, Lamberto Cesari, Herman Chernoff, E. W. Chittenden, Helen M. Clark, Mary Dean Clement, Harvey Cohn, J. B. Coleman, J. A. Colon, B. H. Colvin, E. G. H. Comfort, J. J. Corliss, M. L. Curtis, M. M. Day, John DeCicco, R. F. Deniston, Flora Dinkines, N. J Divinsky, D. G. Duncan, P. S. Dwyer, W. F. Eberlein, H. P. Evans, H. S. Everett, Chester Feldman, W. L. Fields, D. A. Flanders, L. R. Ford, J. S. Frame, Evelyn Frank, Cleota G. Fry, R. E. Fullerton, M. P. Gaffney, B. A. Galler, Murray Gerstenhaber, H. A. Giddings, J. K. Goldhaber, C. C. Goldman, Michael Golomb, R. D. Gordon, R. N. Goss, M. J. Gottlieb, S. H. Gould, L. M. Graves, V. G. Grove, John Gurland, M. M. Gutterman, Marshall Hall, P. R. Halmos, P. C. Hammer, W. L. Hart, H. D. Hartstein, M. Agnes Hatke, Ruth Heinsheimer, R. G. Helsel, Fritz Herzog, G. F. Hewitt, E. H. C. Hildebrandt, T. H. Hildebrandt, G. P. Hochschild, R. E. von Holdt, D. L. Holl, W. A. Howard, C. C. Hsiung, S. P. Hughart, H. K. Hughes, Ralph Hull, P. E. Irick, W. E. Jenner, Meyer Jerison, S. M. Johnson, R. V. Kadison, G. K. Kalisch, L. H. Kanter, Leo Katz, J. B. Kelley, L. M. Kelly, W. M. Kincaid, E. C. Kleinhans, L. A. Knowler, Jacob Korevar, H. G. Landau, E. P. Lane, Leo Lapidus, Sim Lasher, C. G. Latimer, J. S. Leech, K. B. Leisenring, F. C. Leone, D. J. Lewis, O. I. Litoff, A. J. Lohwater, F. W. Lott, W. S. Loud, A. W. McGaughey, Saunders MacLane, W. G. Madow, Morris Marden, A. M. Mark, Kenneth May, D. M. Merriell, H. L. Meyer, Josephine M. Mitchell, J. T. Moore, C. W. Moran, M. L. Mousinho, Leopoldo Nachbin, Zeev Nahari, W. J. Nemerever, K. L. Nielsen, R. Z. Norman, E. P. Northrop, F. S. Nowlan, Rufus Oldenburger, H. W. Oliver, Daniel Orloff, E. H. Ostrow, G. K. Overholtzer, Gordon Pall, Katharine Parks, Mary H. Payne, Marilia C. Peixoto, M. M. Peixoto, George Piranian, G. B. Price, L. E. Pursell, A. L. Putnam, Gustave Rabson, Tibo Rado, G. Y. Rainich, O. W. Rechard, W. T. Reid, Haim Reingold, P. R. Rider, R. A. Roberts, W. G. Rosen, I. A. Rosenberg, Arthur Rosenthal, J. M. Sacks, Hans Samelson, L. J. Savage, R. H. Scherer, O. F. G. Schilling, W. T. Scott, I. E. Segal, B. R. Seth, M. Anice Seybold, M. E. Shanks, A. S. Shapiro, Jack Sherman, J. H. Siedband, G. F. Simmons, I. M. Singer, M. L. Slater, M. F. Smiley, G. W. Smith, Elizabeth S. Sokolnikoff, E. H. Spanier, G. L. Spencer, M. D. Springer, W. L. Stamey, B. M. Stewart, A. F. Svoboda, R. L. Swain, P. M. Swingle, J. V. Talacko, R. M. Thrall, E. A. Trabant, E. F. Trombley, C. H. Tyler, Eugene Usdin, F. A. Valentine, Bernard Vinograde, D. R. Waterman, H. F. Weinberger, L. M. Weiner, M. E. Wescott, J. B. Wright, L. C. Young, J. W. T. Youngs, R. K. Zeigler, Daniel Zelinsky, Antoni Zygmund.

Sessions for contributed papers were held on Friday at 10:45 A.M. and 3:30 P.M., and on Saturday at 10:30 A.M. and 11 A.M. The last session was held jointly with the Institute of Mathematical Statistics. Presiding officers for the various sessions were Professors Marshall Hall, Saunders MacLane, G. B. Price, and G. Y. Rainich. Dr. John Gurland presided over the joint sessions with the Institute of Mathematical Statistics.

At 2:00 P.M. Friday, by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor Hans Samelson, of The University of Michigan, delivered an address entitled *Topology of Lie groups*. Professor A. A. Albert, of the University of Chicago, was the presiding officer.

There was a tea for visiting members and their guests held in the lounge of Eckhart Hall on Friday afternoon.

Abstracts of papers presented follow below. Abstracts whose numbers are followed by the letter "*t*" were read by title. Paper number 357 was read by Professor Buck, paper number 359 by Mr. Klein, paper number 364 by Professor Piranian, and paper number 370 by Mr. Stamey. Mr. Luke was introduced by Professor Eric Reissner, Dr. Zahorski, Mr. Calderon and Mr. Klein by Professor Antoni Zygmund.

Algebra and Theory of Numbers

341. A. A. Albert: *A theory of power-associative commutative algebras.*

Let A be a commutative power-associative ring whose characteristic is prime to 30 and suppose that the equation $2x=a$ has a unique solution x in A for every a of A. We show that if A is simple and contains a pair of orthogonal idempotents whose sum is not the unity quantity of A then A is a Jordan ring. The result may be applied to commutative power-associative algebras A over a field F of characteristic not 2, 3, or 5. Every simple algebra not a nilalgebra has a unity quantity e and we may extend the center F of A to a field K such that $e=e_1+ \cdots +e_t$ for pairwise orthogonal idempotents e_i of A_K. The maximal value of t is called the degree of A and we show that if $t>2$ then A is a Jordan algebra. All simple Jordan algebras of degree $t>2$ are shown to be classical Jordan algebras, that is, the same type of algebras that arise when F has characteristic zero. Define the radical N of any power-associative commutative algebra A of characteristic prime to 30 to be the maximal nilideal of A. Then if e is a principal idempotent of A we show that $A_e(1/2)+A_e(0)$ is contained in N. Thus every semisimple A has a unity quantity and we show that A is uniquely expressible as the direct sum of simple algebras with unity quantities. Our theory thus includes the theory of Jordan algebras of characteristic $p\neq 2, 3, 5$. (Received March 20, 1950.)

342*t*. Rheinhold Baer: *Free mobility and orthogonality.*

Suppose that V is an n-dimensional vector space over the [not necessarily commutative] field F, and that P is a domain of positivity of F. If $b_1, \cdots, b_k$ are independent elements in V, then the subsets $\sum_{j=1}^{i-1} Fb_j+Pb_i$, for $i=1, \cdots, k$, of V form an i-dimensional chain of incident sub-half-spaces of V. The problem is to characterize the groups of linear transformations of V which are simply transitive on the totality of these i-dimensional chains of incident sub-half-spaces of V. This problem is solved under the additional hypotheses that $1<i$ and $2<n$. (Received February 2, 1950.)

343t. J. R. Brown and A. B. Showalter: *On a problem connected with the theory of the Riemann zeta-function.*

In his paper (*On some approximate Dirichlet-polynomials in the theory of the zeta-function of Riemann*, Det Kgl. Danske Videnskabernes Selskab. vol. 24 (1948)) Turán proved that if there is an n_0 such that for $n>n_0$ the partial-sums $U_n(s)=1/1^s+1/2^s+\cdots+1/n^s$ do not vanish in the half-plane $R(s)>1$, then Reimann's hypothesis concerning the roots of the zeta-function is true. Turán also discussed the cases for $n=1, 2, 3, 4, 5$ and showed that $U_n(s)\neq 0$ for these values of n. In this note the authors prove $U_n(s)\neq 0$ for $n=6$ by showing the still stronger result: $R[U_6(s)]>0$ for $R(s)>1$. (Received March 13, 1950.)

344t. F. Marion Clarke: *On the factorization of polynomials in n variables.* IV.

The theorem of the author's Bull. Amer. Math. Soc. Abstract 55-7-399 concerning the uniqueness of the splitting field of polynomial in n variables over a coefficient field of characteristic zero is generalized for polynomials over fields of any characteristic. (Received March 17, 1950.)

345. Nathan Divinsky: *Power-associative crossed extension algebras.*

This paper is a study of power-associative crossed extension algebras, that is, algebras built from an algebra A, a group of nonsingular linear transformations G of A, a subset H of G, and a set of elements g of A. These algebras include crossed product algebras and we first show that a power-associative crossed product is associative. When A is a simple associative algebra over a field of characteristic not 2, we show that the power-associative crossed extension C is either associative or is an extended Cayley algebra. Then we show that if A is a G-simple semi-simple associative algebra over a field of characteristic not 2 and if the identity is the only semi-identity transformation in G, then either C is associative or A is simple when the above result holds. In proving these results we show that every automorphism T of a simple noncommutative ring with minimum condition such that $x\cdot x^T=x^T\cdot x$ for every x must be the identity, and we extend this to semi-simple rings. Finally we generalize the definition of crossed products to include Jordan algebras and find sufficient conditions for these generalized crossed products to be simple Jordan algebras. (Received April 24, 1950.)

346. D. G. Duncan: *Note on a formula by Todd.* Preliminary report.

In a recent paper (Cambridge Philosophical Society, vol. 45, part 3) Todd has given the following formula: $\{m\}\otimes S_n=\sum_\sigma \theta_\sigma\{\sigma\}$. Here $\otimes$ denotes Littlewood's "new" or induced multiplication of S functions. In this note a new characterization is given to the coefficients θ_σ and a short method provided for determining them in terms of the removal of hooks from the Young diagram associated with the partition $[\sigma]$. θ_σ is shown to be zero if the removal of hooks of length n leaves a core and otherwise is plus or minus one according as m plus the sum of the heights of the removed hooks is even or odd. (Received April 24, 1950.)

347. J. K. Goldhaber: *Conditions ordering the characteristic roots of matrices.*

Let $\mathfrak{A}$ be a subalgebra of a total matric algebra of order n^2 over an algebraically closed field $\mathfrak{F}$. Suppose that the identity matrix is in $\mathfrak{A}$. Let $\lambda_{ij}, j=1, \cdots, n$, denote the characteristic roots of $A_i \in \mathfrak{A}$. The following three properties are shown to be equivalent. I: The characteristic roots of every pair of matrices $A_1, A_2 \in \mathfrak{A}$ may be so ordered that the characteristic roots of A_1+A_2 are $\lambda_{1j}+_{2j}$; II: The characteristic roots of every finite set of matrices $\{A_i\}_{i=1}^k \in \mathfrak{A}$ may be so ordered that the characteristic roots of $\sum_{i=1}^k a_i A_i$ are $\sum_{i=1}^k a_i \lambda_{ij}$ for all $a_i \in \mathfrak{F}$; III: Let $f(x_1, \cdots, x_k)$ be any polynomial. The characteristic roots of every finite set of matrices $\{A_i\}_{i=1}^k \in \mathfrak{A}$ may be so ordered that the characteristic roots of $f(A_1, \cdots, A_k)$ are $f(\lambda_{1j}, \cdots, \lambda_{kj})$. This ordering is the same for every f. In the proof of this equivalence the following mapping theorem is used: Let $\mathfrak{A}$ and $\mathfrak{B}$ be linear algebras over F. Let Φ be a mapping of $\mathfrak{A}$ onto $\mathfrak{B}$ satisfying (a) $\Phi(e)=e'$ where e and e' are the units of $\mathfrak{A}$ and $\mathfrak{B}$ respectively, (b) $\Phi(\sum_{i=1}^k a_i A_i)=\sum_{i=1}^k a_i \Phi(A_i)$, and (c) if $\prod_{i=1}^k A_i=0$ then $\prod_{i=1}^k \Phi(A_i)=0$. Then Φ is a homomorphism of $\mathfrak{A}$ onto $\mathfrak{B}-\mathfrak{N}$, where $\mathfrak{N}$ is the radical of $\mathfrak{B}$. (Received March 13, 1950.)

348. J. B. Kelly: *A closed set of algebraic integers.*

An algebraic integer, θ, will be said to belong to class S_1 if all conjugates of θ, save θ itself, lie inside the unit circle and to class S_2 if all conjugates of θ, save θ and one additional conjugate, θ', lie inside the unit circle. The subclass of real elements of S_2 will be denoted by S_2', the subclass of non-real elements by S_2''; if $\theta \in S_2''$, then $\theta'=\bar{\theta}$. Salem (Duke Math. J. vol. 11 (1944) pp. 103–108; Duke Math. J. vol. 12 (1945) pp. 153–172) proved that S_1 is a closed set. It is shown here that although the sets S_2, S_2', and S_2'' are not closed, the set $S_1 \cup S_2''$ is closed. Further, $S_1 \cup S_2''$ has no limit points on the unit circle. (Received April 24, 1950.)

349. D. J. Lewis: *Cubic homogeneous equations in p-adic number fields.*

It is shown that every cubic homogeneous polynomial equation $f(x_1, x_2, \cdots, x_n)=0$ with coefficients in a p-adic number field has a nontrivial integral solution in the field of the coefficients provided $n \geqq 10$. Simple examples of equations having $n \leqq 9$ which possess only the trivial solution are given. Thus ten is the minimum number of variables necessary to insure the existence of nontrivial solutions. (Received March 20, 1950.)

350t. A. R. Schweitzer: *Grassmann's extensive algebra and modern number theory.*

Grassmann's algebra is associated with modern number theory through the "algebraic keys" (clefs algébriques) of A. Cauchy (C. R. Acad. Sci. Paris vol. 36) and through researches of E. Lasker. In this paper the author discusses Grassmann's theory in the light of the following articles by Lasker: (I) *An essay on the geometrical calculus*, Proc. London Math. Soc. vol. 28 (1896–1897) p. 217; (II) Ibid. p. 500; (III) *A new method in geometry*, Amer. J. Math. vol. 30 (1908) p. 65; (IV) *Zur Theorie der Moduln und Ideale*, Math. Ann. vol. 60 (1905) p. 20. (I) constitutes, in effect, a survey of Grassmann's theories of 1844 and 1862 as interpreted by Lasker in terms of algebraic identities, modern algebra, and projective geometry. (II) is construed by the author as a logical continuation of (I) and as a means of transition from (I) to (III); (II) and (III) together are regarded as a connecting bond between Grassmann's Ausdehnungslehre and modern number theory represented by (IV). The latter article constitutes, as is well known, a development of Kronecker's modular systems, that is, ideals of polynomial rings. Reference is made to W. Krull, *Idealtheorie*, Berlin, 1935. (Received March 13, 1950.)

351*t*. A. B. Showalter: *On a problem connected with the extended Riemann hypothesis.*

Let $\chi(n)$ denote a complex character (mod k) and $L(s, \chi) = \sum_{n=1}^{\infty}\chi(n)/n^s$ for $R(s) > 0$. It has been conjectured ("extended Riemann hypothesis") that if $L(s, \chi) = 0$ for complex s [with $R(s) > 0$], then $R(s) = 1/2$. However, it has not even been proved that $L(s, \chi) \neq 0$ for $s > 0$. The latter would be true if (1) $R[L(s, \chi)] > 0$ for $s > 0$. Chowla has conjectured that (1) is true. In this paper the author finds by numerical calculations that (1) is true in a large number of special cases. The author also finds that (2) $R[L(1, \chi)] > 0$ for all complex χ(mod k) where $k \leqq 37$. To prove (1) in special cases the author uses the method described in Acta Arithmetica (vol. 1 (1936) pp. 115–116). To prove (2) he also uses two new methods devised by the author. (Received March 13, 1950.)

352*t*. A. B. Showalter: *Remark on partial sums of L-series.*

Let $f(m \cdot n) = f(m)f(n)$, $f(1) = +1$, and let each $f(n) = +1$, -1, or 0. Let $\lambda(n)$ denote Liouville's number theoretic function. The author proves that if $\sum_{n=1}^{k}\lambda(n)/n > 0$ for all $k \leqq i$, then $\sum_{n=1}^{i} f(n)/n \geqq \sum_{n=1}^{i} \lambda(n)/n$. (Received March 13, 1950.)

353. Bernard Vinograde: *Hermitean classification of matrices.* Preliminary report.

The space M of unitary vectors X such that $AX = A^*X$ for a complex matrix A will be called the hermitean domain of A. A subspace $M_0 \subset M$ is invariant if and only if it completely reduces A unitarily. The usual theory of a Hermitean A is readily modified and extended. Any ring contained in the set of matrices whose hermitean domains contain M is completely reduced by M and easily characterized. (Received March 15, 1950.)

ANALYSIS

354*t*. R. P. Agnew: *Mean values and Frullani integrals.*

Using results of Iyengar (Proc. Cambridge Philos. Soc. vol. 37 (1941) pp. 9–13) and Agnew (Duke Math. vol. 9 (1942) pp. 10–19) or, alternatively, of Ostrowski (Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) pp. 612–616) and a theorem on mean values of functions, we obtain the following incisive theorem involving Frullani integrals and the familiar mean values of $f(x)$ at 0 and at ∞. Let $f(t)$ be Lebesgue integrable over each finite interval $0 < m \leqq t \leqq M < \infty$. If the Frullani integral in the left member of (*) $\int_0^\infty t^{-1}[f(at)-f(bt)]dt=\lambda[\lim_{x\to\infty} x^{-1}\int_0^x f(t)dt-\lim_{x\to 0} x^{-1}\int_0^x f(t)dt]$ exists for each $\lambda=\log (a/b)$ in a set of positive measure, then the mean values in the right member exist and (*) holds for each pair of positive numbers a and b. On the other hand if the mean values in the right member of (*) exist, then the left member exists and (*) holds for each pair of positive numbers a and b. (Received April 11, 1950.)

355*t*. E. F. Beckenbach: *Generalized convex curves.*

In a domain D of the plane, let there be given a family $\{C\}$ of Jordan arcs C each separating D into two components, and such that for each pair of distinct points A and B of D there is exactly one C_{AB} of $\{C\}$ passing through both A and B. Then a Jordan arc K in D shall be said to be $\{C\}$-convex to one of its sides s provided that, for each pair of points A and B of K, the arc between A and B, of the above C_{AB}, lies on the side of K opposite to s. Properties of the family $\{C\}$ and of these generalized convex curves, analogous to known properties of generalized convex functions, are investigated; in particular, generalized convex regions and their boundaries are defined and discussed. (Received March 10, 1950.)

356. L. D. Berkovitz: *Spherical summation and localization of double trigonometric series.* Preliminary report.

Let $U=\sum a_{mn}e^{i(mx+ny)}$ be a double trigonometric series with coefficients $a_{mn}=o(((m^2+n^2)^\gamma)^{1/2})$, $\gamma\geqq -1$. An operator ∇^{-2} is defined, which on each term of U acts as a formal inverse to the Laplacian. To U, ∇^{-2} is applied a sufficient number of times, and a series is obtained which converges *uniformly* to a function $F(x, y)$. If the function F vanishes on a rectangle R, contained in the square bounded by $x=\pm\pi$, $y=\pm\pi$, then for (x, y) in any rectangle R', contained in R, the series U is uniformly circularly summable (in the sense of Bochner) $(C, \gamma+1)$ to zero. This fact is a consequence of localization formulae which for the case $\gamma=-1$, say, read: $(\sum_{m^2+n^2\leqq R^2} a_{mn}e^{i(mx+ny)}-(1/4\pi^2)\int_0^{2\pi}\int_0^{2\pi}F(u, v)\lambda(u, v)\nabla^2 D_R(u-x, v-y)dudv)\to 0$ where $\lambda(u, v)$ is a sufficiently regular localizing function, and $D_R(x, y)$ is the analogue of the Dirichlet Kernel for circular summation of double Fourier Series. The method of proof uses formal multiplication of series in a manner somewhat analogous to that developed for single series by Rajchman and Zygmund, but applied to circular sums and means. (Received March 15, 1950.)

357. R. P. Boas and R. C. Buck: *Expansions of entire functions in polynomial series.*

J. M. T. Whittaker has obtained conditions under which an entire function has a convergent expansion in a basic series $\sum C_n p_n(z)$ (*Interpolatory function theory*, Cambridge, 1938). One of the authors has observed that in the case of Appell polynomials, convergence may occur for a much larger class of functions, if the coefficients $\{C_n\}$ are chosen differently. In the present paper, a general theory for the expansion of entire functions into convergent or summable series of Appell-type polynomials is obtained. (Received March 16, 1950.)

358. A. P. Calderon and George Klein: *On an extremal problem concerning trigonometrical polynomials.*

This paper is concerned with the following generalization of a theorem of P.

Erdös: suppose $\phi(x)$ is a function defined for non-negative x and satisfies the condition that $(\phi(x)-\phi(0))/x$ be a non-negative non-decreasing function of x, $x \geqq 0$. Then the maximum of the integral $\int_0^{2\pi}\phi(|S'(x)|)dx$ for all real trigonometrical polynomials $S(x)$ of order n, bounded in absolute value by 1, is achieved by the Tchebycheff polynomial cos $(nx+\alpha)$. If ϕ is not a constant function, then among all such polynomials the Tchebycheff polynomial only achieves this maximum. This theorem applies, for example, to nondecreasing functions and the theorem of Erdös referred to above for arc length is obtained by setting $\phi(x)=(1+x^2)^{1/2}$. (Received April 28, 1950.)

359*t*. C. C. Camp: *An expansion associated with an irreducible partial differential equation.* Preliminary report.

The equation $u_x+u_y+\lambda a(x, y)u=0$ for which $a(x, y)$ is not separable into $g(x)+h(y)$ cannot be reduced by Bernoulli's method to a system of ordinary differential equations. The boundary conditions $W_1=\int_0^1[1-u(x, 1)\, v(x, 0)]dx=0$, $W_2=\int_0^1[u(1, y)v(0, y)-1]dy=0$, where $v=1/u$ is a solution of the adjoint equation, likewise cannot be carried over to equivalents for an ordinary boundary value problem unless $a(x, y)$ is properly restricted. Although a formal expansion can be set up, even with a second parameter μ, such that $u=\exp\,[-\lambda H(x, y)+\mu(x-y)]$ and $H_x+H_y=a(x, y)$, no valid expansion exists since it can be shown that the only characteristic value for λ is $\lambda=0$. If $H(x, 1)=H(x, 0)$ and $H(1, y)=H(0, y)$, then λ will have a countably infinite number of characteristic values and the usual convergence proof involving a Green's function and a double contour integral in λ, μ can be carried through. Without the second parameter μ the formal expansion can be shown not valid. This theory is being extended to more than 2 independent variables. (Received March 15, 1950.)

360. W. F. Eberlein: *Decomposition of weak almost periodic functions.* Preliminary report.

Let $x(t)$ $(-\infty<t<\infty)$ be weakly almost periodic (for this concept cf. W. F. Eberlein, Trans. Amer. Math. Soc. vol. 67 (1949) pp. 217–240). Then $x(t)$ admits the unique decomposition $x(t)=x_1(t)+x_2(t)$, where $x_1(t)$ is uniformly almost periodic, and $x_2(t)$ is a weakly almost periodic function all of whose Fourier coefficients vanish. The point spectrum of a weakly almost periodic function is thus determined. Preliminary results on extension of the theorem to arbitrary locally compact Abelian groups are obtained. (Received February 13, 1950.)

361. R. E. Fullerton: *Characterizations of L and C spaces.*

Let X be a Banach space whose unit sphere S is the convex closure of its set of extreme points. Then X is a space of integrable functions or of continuous functions if any one of the following three equivalent conditions hold: (1) If $\|x\|=1$, $x\in F$ where F is a maximal convex subset of the surface of S and S is the convex set determined by F and $-F$. (2) Every extreme point of S is the vertex of a cone C having the property that $S=C\cap(-C)$. (3) If $u\in S$ is an extreme point of S, then for any λ, $0\leqq\lambda<2$, $S\cap((\lambda/2)u+S)=(\lambda/2)u+(1-\lambda/2)S$. (Received March 15, 1950.)

362. L. M. Graves: *Induced linear transformations on spaces of continuous functions.*

Let $\mathfrak{U}$ and $\mathfrak{B}$ be Banach spaces, and $\mathfrak{S}$ a compact metric space. Let $\mathfrak{X}$ and $\mathfrak{Y}$ denote the Banach spaces consisting of all continuous functions on $\mathfrak{S}$ to $\mathfrak{U}$ and to $\mathfrak{B}$, respectively. Let Ω_0 be the space of all linear continuous transformations K mapping $\mathfrak{U}$ *onto* $\mathfrak{B}$. Let $L(K, v) = \text{g.l.b.}\|u\|$ for $Ku = v$, $I(K) = \text{l.u.b.}\ L(K, v)$ for $\|v\| = 1$. By a theorem of Banach, $I(K) < \infty$ on Ω_0. Now let $K(s)$ be continuous on $\mathfrak{S}$ to Ω_0, and let $y = \kappa x$ denote the induced transformation defined by $y(s) = K(s)x(s)$. Then κ maps $\mathfrak{X}$ *onto* $\mathfrak{Y}$, and $I(\kappa) = \max I(K(s))$ on $\mathfrak{S}$. Moreover, if $\mathfrak{S}$ is a linear interval, and $N > I(\kappa)$, then for each y there exists x with $\kappa x = y$ and $\|x(s)\| < N\|y(s)\|$ on $\mathfrak{S}$. In this special case, the proof is made by constructing a sequence of polygonal approximations to a solution $x(s)$. By induction the theorem may be extended to the case when $\mathfrak{S}$ is an interval in n-space, and then to the Hilbert parallelotope. From this result the general case is obtained with the help of a device used by Hestenes. The theorem has numerous applications in securing existence theorems for the solution of functional equations. (Received March 13, 1950.)

363. Josephine M. Mitchell: *A convergence theorem for multiple Fourier series summed by spherical means.*

Let $S_R = \sum_{\nu \leqq R} a_{m_1 \cdots m_k} \exp\ (i(x_1m_1 + \cdots + x_km_k))$ $(\nu = m_1^2 + \cdots + m_k^2)$ be the spherical partial sum of the Fourier expansion of a Lebesgue square integrable function $f(x_1, \cdots, x_k)$ defined on the k-dimensional hypercube $D(-\pi \leqq x_j \leqq \pi, j = 1, \cdots, k)$. The Lebesgue function $(1/(2\pi)^k)\int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} \left| \sum_{\nu \leqq R} \exp i \sum m_j(x_j - y_j) \right| dy_1 \cdots dy_k = O(R^{k/4})$ from which it follows that $S_R = O(R^{k/8})$. Consequently we may prove the theorem that if $\sum (m_1^2 + \cdots + m_k^2)^{k/4} a_{m_1 \cdots m_k}^2 < \infty$, then $\lim_{R \to \infty} S_R$ exists almost everywhere in D. For the cases $k = 2$ and 3 the exponent $k/4$ may be made considerably smaller. The proofs of these statements use the classical methods of Fourier and orthogonal series and represent an improvement over results obtained by different methods (cf, for example, Chandrasekharan and Minakshisundaharan, Duke Math. J. vol. 14 (1947) and Chandrasekharan, Bull. Amer. Math. Soc. vol. 52 (1946)). (Received March 15, 1950.)

364. George Piranian and A. J. Lohwater: *Linear accessibility in Jordan regions.*

A point P on the boundary C of a Jordan region R is said to be *linearly accessible* if there exists a rectilinear segment QP whose points lie interior to R, except for P. There exists a Jordan curve C in the ζ-plane such that, under the mapping $z = f(\zeta)$ of the simply-connected region determined by C onto the unit circle, the set of linearly accessible points of C is mapped into a set of Lebesgue measure zero. The proof is based on the construction of an appropriate Taylor series with large gaps. (Received March 15, 1950.)

365*t*. R. K. Ritt: *Algebraic functions in an Abelian normed ring.* Preliminary report.

Let $K = R(e, a)$ be an Abelian normed ring, $\sigma(a)$ the spectrum of a. This paper examines the relationship between the topology of $\sigma(a)$ and the possibility of solving the equation $u^n = b$, $b \in K$. Applications are made to transformations in a Banach space. (Received March 20, 1950.)

366*t*. I. E. Segal: *Decomposition of operator algebras.* II. *Multiplicity theory.*

Every commutative W^*-algebra (=weakly closed and self-adjoint algebra of bounded linear operators on a Hilbert space) is a direct sum of W^*-algebras of uniform multiplicity, an algebra of the latter type of multiplicity $n>0$ being unitarily equivalent to an n-fold copy of a maximal abelian W^*-algebra. This last algebra is unitarily equivalent to the algebra of all multiplications by bounded measurable functions on L_2 over a suitable measure space. An arbitrary W^*-algebra is determined within unitary equivalence by the Boolean ring (of all measurable sets modulo null sets) associated thereby with each multiplicity. It follows that any commutative W^*-algebra is algebraically isomorphic to a maximal abelian W^*-algebra in a fashion which is weakly bicontinuous and which preserves the operational calculus. The decomposition of W^*-algebras into parts of uniform multiplicity is valid also for non-commutative algebras, and any W^*-algebra of finite uniform multiplicity $n>0$ is unitarily equivalent to an n-fold copy of a W^*-algebra A for which A' is abelian. (Received April 7, 1950.)

367. B. R. Seth: *Some solutions of the wave equation.*

A number of solutions of the two-dimensional wave equation $\Delta^2\phi=\partial^2\phi/\partial t^2$ have been obtained by B. R. Seth, D. G. Christopherson, and P. N. Sharma. In all these solutions the boundary is rectilinear and either ϕ or the normal derivative vanishes over the boundary. In the present paper the boundary condition is taken as ϕ=a constant, and the corresponding forms of ϕ are obtained for a number of boundaries. For an equilateral and some rhombus and pentagonal boundaries ϕ is obtained in a finite number of trigonometrical terms. (Received February 17, 1950.)

368t. Zygmunt Zahorski: *On the first derivative.* (*Sur la première dérivée.*)

This paper is concerned primarily with characterizing five classes of functions in terms of the properties of the sets $[f'(x)>a]$, where $f'(x)$ exists everywhere. In the case where $f'(x)$ is bounded, necessary and sufficient conditions are developed. For the cases where $f'(x)$ is either finite or assumes the values $\pm\infty$, only necessary conditions are developed. The five classes of functions considered comprise a descending sequence intermediate between the first class of Baire with Darboux property and the class of approximately continuous functions. In particular, the following theorem is established: Let $f(x)$ be a function which passes through all mean values and such that, allowing infinite values, $f'(x)$ exists everywhere, with the possible exception of the points of an enumerable set, and $f'(x)\geqq 0$ almost everywhere; then $f(x)$ is continuous and non-decreasing. This theorem is established without making any assumption as to the measurability of $f(x)$. (Received April 24, 1950.)

Applied Mathematics

369. Y. L. Luke: *Some notes on integrals involving Bessel functions.*

In a number of applied mathematics problems, the results depend on the values of $\int_0^x e^{ku}u^\nu Z_n(\lambda u)du$ where $Z_n(u)$ represents any of the Bessel functions of the first three kinds or the modified Bessel functions. The purpose of this note is to develop some recursion formulae and power series expansions, which can be used to compute the above integral. The results are applied to show that $\int_0^x e^u K_0(u)du=xe^x[K_0(x)+K_1(x)]-1$. Thus one obtains an alternative proof that the constant term in the asymptotic expansion of the latter integral is -1. (H. Bateman and R. C. Archibald, *Guide to* *tables of Bessel functions*, MTAC, vol. 1, no. 7, July 1944, p. 226.) Other applications also are presented. (Received March 2, 1950.)

Geometry

370. L. M. Blumenthal and W. L. Stamey: *Characterization of pseudo-$S_{n,r}$ sets.*

A semimetric space S is pseudo-$S_{n,r}$ provided each of its $(n+2)$-tuples is congruently imbeddable in the convexly metrised n-sphere $S_{n,r}$ of diameter πr, but S itself is not. If no two points of a pseudo-$S_{n,r}$ set S have distance πr, it was shown by Blumenthal and Thurman (Amer. J. Math. vol. 62 (1940) pp. 835–854) that for every integer k, the matrix $\|\cos(p_ip_j/r)\|$ $(i, j=1, 2, \cdots, k)$ of each k-tuple $p_1, p_2, \cdots, p_k$ of points of S has (upon multiplication by -1 of appropriate rows and the same numbered columns) every element equal to 1 or $-1/(n+1)$, provided S contains more than $n+3$ points. In this note, the characterization is completed by showing that the restriction that S be without diametral point-pairs may be dropped. The Blumenthal-Thurman theorem remains valid provided S contains more than $2(n+3)$ points, or if S contains $n+k+3$ points $(k>0)$, not more than $k-1$ of its pairs be diametral. (Received March 15, 1950.)

371. H. R. Brahana: *Finite metabelian groups and the lines of a projective four-space.*

The groups dealt with are metabelian (that is, of class 2); their elements are all of order p; their centrals and commutator subgroups coincide; and each is generated by five elements. There is one largest group G, of order p^{15}, with these properties; every other group is a quotient group of G. The cyclic subgroups of the central quotient group of G can be represented on the points of a finite projective four-space X. The lines of X correspond to the commutators of G; the lines of X can be represented on points of a projective nine-space S, these points constitute a locus V of order five and dimension six. The problem of the determination of the groups is the problem of the possible relations of points, lines, planes, and so on, in S to V. This paper determines the 85 groups of order p^{15} to p^{11} (there are 54 of order p^{11}). Canonical forms are given which are immediately translatable into defining relations of the groups. The geometric relations which determine the canonical forms are invariant under collineations in X and translate into properties of the groups which are independent of the generating elements. (Received March 16, 1950.)

372. G. Y. Rainich: *The neglected part of the Riemann tensor.*

A tensor of rank four is considered (in flat Euclidean four-space) which has the algebraic properties of the Riemann tensor and also satisfies the Bianchi equations. On this tensor is imposed the condition that its contracted tensor vanish. The remaining components are arranged into a matrix M of three rows and six columns such that the two halves of it are three-by-three symmetric matrices of zero trace. The transformation properties of these matrices under rotations of the four-space are described by presenting such rotation as product of two elementary rotations (for each of which every vector belongs to an invariant plane) of opposite type. Each of these does not affect one half of M and transforms the other half as a rotation in three-space transforms a symmetric tensor of rank two. The Bianchi relations reduce to a system of differential equations which may be considered as generalizations of the Cauchy-Riemann equations for analytic functions. The equations for each row of M are also the exact Euclidean analogues of Maxwell's equations but the three sets of these Maxwell equations are coupled by having some components in common. (Received March 14, 1950.)

373. J. B. Wright: *Meta-projective geometry.*

A geometry without points and without hyerplanes is formulated as an extension of projective geometry. An equivalence relation produces a dichotomy of elements in which those of one equivalence class are related to those of the other, as points are related to planes. The reflexivity, symmetry, and transitivity of this relation follow from the first four axioms which make use of chains (sequences of elements each joined to the next by incidence). The axiom that every five-chain is open implies that every five-chain can be closed (transitivity of incidence). An infinite-dimensional counter example is eliminated by an axiom guaranteeing the existence of a finite set of equivalent elements without common incident. One says that n elements are independent if the incidence of $n-1$ of them to an element does not imply the incidence of the remaining. The axiom that n independent elements have a unique common incident makes it possible to prove the usual properties of bases and dimensionality of the space. Axioms for n-dimensional projective geometry over a division ring become theorems in our system. In three dimensions the Moebius theorem assures the commutativity of the number system. (Received February 10, 1950.)

Statistics and Probability

374. P. E. Irick: *A geometric method for finding the distribution of standard deviations when the sampled population is arbitrary.* Preliminary report.

For an ordered random sample, $x_1 \leqq x_2 \leqq \cdots \leqq x_n$, chosen from a population, $f(x)$, $a \leqq x \leqq b$, let $r_i = x_{i+1} - x_i \geqq 0$, $i = 1, 2, \cdots, n-1$. Then make the transformation $r_i = -((i-1)/2i)^{1/2} r'_{i-1} + ((i+1)/2i)^{1/2} r'_i$, and call U' the $1/n!$ portion of the r' space bounded by the $n-1$ sphere and hyperplanes, $\sum_{i=1}^{n-1} r_i'^2 = 2ns^2$, $\cdot r'_i = ((i-1)/(i+1))^{1/2} r'_{i-1}$, $i = 1, 2, \cdots, n-1$, where s is the sample standard deviation. The point density in U', $\delta(r')$, is the transform of $\delta(r) = \int_{x_1=a}^{b-\Sigma r_i} f(x_1) f(x_1 + r_1) \cdots f(x_1 + r_1 + \cdots + r_{n-1}) dx_1$. Change to generalized polar coordinates and call U the outer hyperspherical boundary of U' whereon the density is designated by $\delta((2n)^{1/2}s, \phi)$. Then $p(s)$, the probability law for s, is given by $p(s)ds = n! n^{n/2} s^{n-2} ds \int_{\phi_1} \cdots \int_{\phi_{n-2}} \delta((2n)^{1/2}s, \phi) \sin^{n-3}_{\phi_1} \cdots \sin \phi_{n-3} d\phi_{n-2} \cdots d\phi_1$, where arc cos $((n/(n-i)(i+1))^{1/2} \leqq \phi_i \leqq$ arc cos $[((i-1)/(i+1))^{1/2}/\tan \phi_{i-1}]$, $i = 1, 2, \cdots, n-2$, whenever b is infinite. The distribution of sample range is readily found in U' and is expressible in the same form as $p(s)$ with the same limits of integration. When b is finite, the complete integral holds only for $0 \leqq s \leqq (b-a)/(2n)^{1/2}$, there being $n^2/4$ connected arcs in $p(s)$ if n is even, and $(n^2-1)/4$ arcs if n is odd. The axes are rotated to give relatively simple formulas for $p(s)$ when $n \leqq 4$, the case of $n = 5$ also being discussed. The method readily produces previously reported results for $p(s)$. In the application of the method, particular attention has been paid to the Type III and polynomial Type I populations. The density function provides much information concerning the form of $p(s)$ for various populations, and contours of constant δ in U' are of theoretical interest. (Received March 15, 1950.)

375. W. M. Kincaid: *Analysis of a one-person game.* Preliminary report.

The problem of allocation of supplies is one which arises in many military and economic connections. The present report discusses a game constructed as a model of a simple situation of this type. The player is given a supply of cards, and receives payments for giving these up when certain random events occur during the period of play. The optimal strategy, which maximizes the expected value of these payments, is governed by certain critical times such that the player's response to a particular event depends on whether it occurs before or after one of these times. (Received March 3, 1950.)

376. W. S. Loud: *Probability of a correct result with a certain rounding-off procedure.*

Consider the problem of the addition of n numbers expressed in the base B of numeration. Supposing each number known to arbitrary accuracy, to obtain the sum accurate to k places, one may round off each number to $(k+1)$ places, add, and round the sum to k places. If the numbers are assumed uniformly distributed, the probability that the above procedure gives the correct result may be found explicitly by use of characteristic functions. If the base B is odd, the result is $2(\pi B)^{-1}\int_0^\infty \sin^{n-1} u \cdot \sin^2 Bu\, u^{-n-1}du$, and if the base B is even, $2(\pi B)^{-1}\int_0 \sin^{n-1} u \sin^2 Bu \cos u\, u^{-n-1}du$. Both formulas have the asymptotic formula $6^{1/2}B(\pi n)^{-1/2}$ as n becomes infinite. (Received March 13, 1950.)

377. P. R. Rider: *The distribution of the quotient of ranges in samples from a rectangular population.*

The distribution of the quotient of two independent, random samples from a continuous rectangular population is derived. The distribution is independent of the population range and can be used to test the hypothesis that two samples came from the same rectangular population, just as the distribution of the variance ratio is used to test whether two samples came from the same normal population. (Received March 13, 1950.)

Topology

378*t*. E. A. Michael: *Topologies on subsets.*

Two topologies on the collection 2^X of closed subsets of a topological space X are studied. The finite topology (L. Vietoris, Monatshefte für Mathematik und Physik, 1922) is the coarsest topology on 2^X in which $\{E\in 2^X \mid E\subset A\}$ is closed if A is closed and open if A is open. The uniform topology (N. Bourbaki, *Topologie générale*, chap. 2, p. 97), defined on 2^X if X is uniform, generalizes Hausdorff's topology for a metric X. It is shown that these topologies agree on $C(X)$, the collection of compact subsets of X, for any uniform X; they agree on all of 2^X if X is normal, and if the uniform structure is induced by the Stone-Čech compactification. Various properties of 2^X and $C(X)$, known for the Hausdorff metric topology, are extended. Among new results, the following is typical: Let X be a topological space with topology T. Then a necessary and sufficient condition for the existence of a continuous function from 2^X to X which maps each element of 2^X into a point of itself is that there exist a linear ordering on X such that the order topology is coarser than T and such that every closed (T) subset of X has a first element. (Received March 16, 1950.)

379. P. M. Swingle: *The closure of types of connected sets.*

Using where necessary the hypothesis of the continuum and Zermelo's axiom, it is shown in any connected domain, D, in the plane there exist subsets W, B, F, and T such that W is widely connected, B is a biconnected set without dispersion point, F is a finitely-containing connected set, and both T and $D-T$ are indecomposable connexes—and D, W, B, F, T, and $D-T$ all have the same closure. (Received March 8, 1950.)

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