

seems to be some danger that a reader might get lost in the details of specific transformation equations and long-winded discussions of apparent paradoxes. Here are a few examples of the method of presentation: Minkowski's space-time geometry is introduced in the sixth and last section of the long chapter on special relativity, whereas the proof of the Lorentz invariance of Maxwell's equations is given in section 5. The chapter on tensors does not include any discussion of covariant differentiation or the proof of the tensor character of the curvature tensor. In the chapter on general relativity, the Schwarzschild line element is given without derivation. On the other hand, a detailed calculation is made to show that certain radial lines are geodesics; here a simple symmetry argument would have sufficed.

The author attaches some importance to the possibility of retaining a "world ether" consisting of "tiny ether atoms." This point of view is perhaps old fashioned at a time when most physicists would dispute the usefulness of introducing a mechanistic model for its own sake.

For purposes of reference the value of the book is impaired by the very large number of abbreviations. An index of these would have been useful.

A. SCHILD

*The theory of algebraic numbers.* By Harry Pollard. (Carus Mathematical Monographs, no. 9.) Buffalo, The Mathematical Association of America; New York, Wiley, 1950. 12+143 pp. \$3.00.

For many years no suitable treatment of algebraic number theory has been available in English. The present monograph, presenting a concise and careful exposition of the elements of the classical theory, is thus most welcome.

The introductory material covers the unique factorization of the rational integers and of the Gaussian integers  $m+ni$ , together with the (little) Fermat theorem in both cases. The familiar example of nonunique factorization for the integers  $m+n(-5)^{1/2}$  is then presented. Then algebraic numbers are defined in general, and the (constructive) existence of transcendental numbers is demonstrated. The elementary properties of algebraic number fields and of algebraic integers are then developed. For the fundamental theorem of ideal theory (every ideal can be represented uniquely as a product of prime ideals) both the classical proof and the Ore modification of the Noether-Krull axiomatic proof are presented. Among the consequences is the proof that a rational prime is unramified in a field if it does not divide the discriminant of the field.

The definition of an ideal is motivated by a tentative description

of an ideal in a number field  $K$  as the set of all integral multiples in  $K$  of some algebraic integer which does not necessarily lie in  $K$ . In Chapter 10 it is demonstrated that every ideal can in fact be so described, the proof depending on the finiteness of the class number of a field. The application of algebraic number theory to the Fermat conjecture ( $x^p + y^p = z^p$ ) is then discussed, on the basis of previous illustrative analysis of the cyclotomic field generated by the  $p$ th roots of unity. The last chapter presents the Minkowski proof of the Dirichlet theorem on the units of a number field. The material is well arranged, with frequent appropriate examples, and is self-contained except for the fundamental theorem of algebra, the theorem on symmetric functions, and the basic facts about simultaneous homogeneous linear equations.

The reviewer regrets that the author did not take a little additional space to point out the fascinating relations between the specific concepts arising in algebraic number theory and the more general concepts of algebra. Thus, the fact that an ideal is precisely the kernel of a homomorphism is not mentioned. A field means exclusively a field of numbers; hence the revealing circumstance that the congruence classes of integers modulo a prime ideal constitute a finite field cannot be brought out. In keeping with an ancient tradition in number theory, the word "group" is never mentioned, in the midst of numerous arguments of a group-theoretic character. The fact that the ideal classes form a group is thus not clearly stated, and the familiar argument that a subgroup of a free abelian group is free appears in disguise at least twice (for units, and for bases of an ideal).

This book provides a clear and elementary introduction to its subject, in keeping with the purpose of the Carus Monographs. If supplemented as indicated, it would be of use in elementary graduate courses.

SAUNDERS MACLANE

#### BRIEF MENTION

*Collected mathematical papers.* By G. D. Birkhoff. Vol. 1, 43+754 pp.; vol. 2, 8+983 pp.; vol. 3, 8+897 pp. New York, American Mathematical Society, 1950. \$18.00.

In addition to the collected papers, volume 1 contains obituaries by R. E. Langer, O. Veblen, and M. Morse, and volume 3 contains a list of Birkhoff's publications. The obituary by Morse, reproduced from Bull. Amer. Math. Soc. vol. 52 (1946) pp. 357-391, contains a detailed review of Birkhoff's work.

*Contributions to Fourier analysis.* By A. Zygmund, W. Transue, M.