tion of transformation groups expressing various symmetries with the integration theory of partial differential equations is discussed.

J. J. STOKER

Introduction à la géométrie non euclidienne par la méthode élémentaire. By G. Verriest. Paris, Gauthier-Villars, 1951. 8+193 pp. 1000 fr.

This exposition begins with the foundations of Euclidean geometry: Hilbert's axioms of incidence, order, congruence, and parallelism. Setting aside the axiom of parallelism, the author develops a number of theorems belonging to both Euclidean and hyperbolic geometry. He takes care to avoid any appeal to intuition; for example, he proves in detail that the sum of two supplementary angles is equal to the sum of two right angles. This part of the work, making no assumption of continuity, culminates in the "second theorem of Legendre": If, in at least one triangle, the angle-sum is equal to two right angles, the same holds for every triangle. In Chapter IV, Dedekind's axiom of continuity is used to prove the axiom of Archimedes and the "first theorem of Legendre": The angle-sum of a triangle cannot exceed two right angles. The author mentions that this was anticipated by Saccheri, and that it would not hold without continuity [Dehn, Math. Ann. vol. 53 (1900) pp. 436–439]. Chapter V establishes the equivalence, in the presence of the other axioms, of three possible formulations of the Euclidean axiom: Euclid's own Postulate V, the unique parallel through a given point to a given line, and the angle-sum of a triangle being equal to two right angles. It follows that all three are contradicted in hyperbolic geometry.

The next three chapters are on Hilbert's treatment of area and its connection with angle-sum. Chapter IX deals with simple properties of parallels and ultraparallels in the hyperbolic plane. On p. 142 the author omits, as being "assez compliquée," the proof of the existence of a common parallel line to two given rays [Hilbert, *Grundlagen der Geometrie*, Leipzig, 1913, p. 151; Coxeter, *Non-Euclidean geometry*, Toronto, 1947, p. 205]. He excuses the omission by declaring that this result will not be required later; but he actually uses it on p. 150 and again on p. 156, each time assuring the reader that it will not be needed any more! There is a very readable account of circles, horocycles, and hypercycles, with a short paragraph on the extension to three dimensions.

Elliptic geometry is relegated to a single chapter at the end. The statement that every two coplanar lines intersect is shown to imply that each line of the plane has one or two absolute poles. The two alternative hypotheses are carefully examined, with appropriate modifications of the axioms of incidence and order. There is a neat treatment of Clifford parallels, and a proof that nothing like them can occur in hyperbolic geometry.

The book is well written, simple yet rigorous. On the other hand, it is slightly old-fashioned, long-winded in some places, and a little too much dominated by Hilbert. There is no history or bibliography; the names of Gauss and Bolyai, Cayley and Klein are seldom (if ever) mentioned.

H. S. M. COXETER

Tensor calculus. By J. L. Synge and A. Schild. (Mathematical Expositions, no 5.) University of Toronto Press, 1949. 12+324 pp. $6.00.

This book is an outgrowth of a series of lectures delivered by Professor Synge at the University of Toronto, Ohio State University, and Carnegie Institute of Technology. It is a general brief introduction to the calculus of tensors and its applications without the usual historical development of the subject. A short bibliography of some of the leading texts on the subject is given on page 319 and an index on pages 321–324.

There are eight chapters. The first four deal with the usual concepts of tensors, Riemannian spaces, Riemannian curvature, and spaces of constant curvature. The next three chapters are concerned with applications to classical dynamics, hydrodynamics, elasticity, electromagnetic radiation, and the theorems of Stokes and Green. In the final chapter, an introduction is given to non-Riemannian spaces including such subjects as affine, Weyl, and projective spaces. There are two appendices in which are discussed the reduction of a quadratic form and multiple integration. At the conclusion of each chapter, a summary of the more important formulas is given and also a set of exercises is included to illustrate the material of the chapter.

In the first two chapters the authors discuss the concepts of absolute tensors and Riemannian geometry. The equations of the geodesics are derived by the methods of the calculus of variations. After showing that the expressions for the covariant derivative of a vector form a tensor, the authors define parallel displacement of a vector along a curve by the vanishing of the absolute derivative. The elegant geometrical treatment of Levi-Civita of parallel displacement is not mentioned. Among other subjects treated are the Serret-Frenet formulas and the curvatures of a curve in a general Riemannian space. In the third chapter, the Riemannian curvature tensor is introduced by means of the commutation rule for the covariant second deriva-