modifications of the axioms of incidence and order. There is a neat treatment of Clifford parallels, and a proof that nothing like them can occur in hyperbolic geometry.

The book is well written, simple yet rigorous. On the other hand, it is slightly old-fashioned, long-winded in some places, and a little too much dominated by Hilbert. There is no history or bibliography; the names of Gauss and Bolyai, Cayley and Klein are seldom (if ever) mentioned.

H. S. M. COXETER

Tensor calculus. By J. L. Synge and A. Schild. (Mathematical Expositions, no 5.) University of Toronto Press, 1949. 12+324 pp. $6.00.

This book is an outgrowth of a series of lectures delivered by Professor Synge at the University of Toronto, Ohio State University, and Carnegie Institute of Technology. It is a general brief introduction to the calculus of tensors and its applications without the usual historical development of the subject. A short bibliography of some of the leading texts on the subject is given on page 319 and an index on pages 321–324.

There are eight chapters. The first four deal with the usual concepts of tensors, Riemannian spaces, Riemannian curvature, and spaces of constant curvature. The next three chapters are concerned with applications to classical dynamics, hydrodynamics, elasticity, electromagnetic radiation, and the theorems of Stokes and Green. In the final chapter, an introduction is given to non-Riemannian spaces including such subjects as affine, Weyl, and projective spaces. There are two appendices in which are discussed the reduction of a quadratic form and multiple integration. At the conclusion of each chapter, a summary of the more important formulas is given and also a set of exercises is included to illustrate the material of the chapter.

In the first two chapters the authors discuss the concepts of absolute tensors and Riemannian geometry. The equations of the geodesics are derived by the methods of the calculus of variations. After showing that the expressions for the covariant derivative of a vector form a tensor, the authors define parallel displacement of a vector along a curve by the vanishing of the absolute derivative. The elegant geometrical treatment of Levi-Civita of parallel displacement is not mentioned. Among other subjects treated are the Serret-Frenet formulas and the curvatures of a curve in a general Riemannian space. In the third chapter, the Riemannian curvature tensor is introduced by means of the commutation rule for the covariant second derivative. Parallel displacement around a closed circuit is discussed, and various formulas for the Riemannian curvature are obtained. In particular, it is proved that parallel displacement is independent of the path if and only if the space is flat. In the fourth chapter, the important theorem of Schur is established, namely, if the Riemannian curvature is locally constant, then it is constant throughout the whole space. The remaining subject matter is devoted to spaces of constant curvature and flat spaces. In a space of positive constant curvature, the geodesics through a point P meet again in a point P'. The space is antipodal (spherical) or polar (elliptic) according as $P' \neq P$ or $P' = P$. Flat spaces are studied by means of homogeneous coordinates which are cartesian coordinates for flat spaces with positive definite quadratic form. The theory of cartesian tensors is developed. In this connection, the special theory of relativity is briefly discussed.

In the fifth chapter, the dynamics of a particle are studied. The Lagrangian equations and, for a conservative field of force, the potential and action integral including Hamilton's principle of least action, are developed. Elsewhere these concepts have been extended so as to include not only dynamical trajectories, but also brachistochrones, catenaries, velocity systems, and other families of curves in a given field of force. Also studied are the equations of motion of a rigid body. In the sixth chapter, the Lagrangian method and the Euler method of describing the motion of a fluid are developed. After establishing the equation of continuity, the authors give the general equations of motion of a perfect fluid in general curvilinear coordinates. For an irrotational fluid, the integral of Bernoulli is found. In elasiticity, the stress and strain tensors are developed, and the equations of motion of a continuous medium are given. In the theory of electromagnetic radiation, Maxwell's equations are written in general curvilinear coordinates. Special types of solutions are given and various interesting geometrical properties of these solutions are obtained. The sixth chapter concludes with a brief discussion of the general theory of relativity.

In the seventh chapter, the authors discuss relative tensors and the nonmetrical concept of extension, originally due to Grassmann. A generalized Stokes theorem is established without the use of a metric. Finally a general form of Green's theorem is obtained. In the eighth and final chapter, an introduction to non-Riemannian geometries is given. After a discussion of a general affine space and the affine curvature tensor, the authors study the gauge transformations in a Weyl space and the gauge invariants. The final subject consists of projective geometry and the Weyl projective curvature tensor.

This tensor vanishes identically for a projective space of two dimensions. For spaces of three or more dimensions, the Weyl projective curvature tensor vanishes if and only if the given affine space is projectively equivalent to a flat space.

Although the book seems to lack geometrical motivation, the material is presented in a very understandable fashion. The ideas and concepts are given very concisely and thus a wide range of subjects is covered. This book is a very worthwhile introduction to the subject for a beginning student of the absolute differential calculus and its applications.

JOHN DECICCO

BRIEF MENTION

The hodograph method in gas dynamics. By A. G. Ghaffari. Teheran, Taban, 1950. 129 pp., 19 figs.

This book gives an introductory account of one of the most interesting methods in gas dynamics, the hodograph method. Particular emphasis has been given to approximate methods (Chapters IV, V, VII), including some of the author's own work. Chaplygin's method and its current developments are discussed briefly in Chapter III, while Chapter VI gives a rather detailed account of Bergman's method. A clear presentation of the standard material in gas dynamics is given in Chapters I, II, III, and VIII. The book contains a preface by Professor G. Temple which concludes with the statement: "This book can be confidently recommended to students of aerodynamics who desire an elementary survey of the theory and use of the Hodograph method."

C. C. LIN

Gelöste und ungelöste mathematische Probleme aus alter und neuer Zeit. By Heinrich Tietze. Munich, Biederstein, 1949; vol. 1, 20+256 pp.; vol. 2, 4+305 pp.

This is one of the deepest and at the same time the most charming of the popular books on mathematics that I have ever seen. In two moderate sized volumes Tietze presents fourteen lectures, on topics as diverse as digital representations of integers, distribution of primes, properties of geodesics on surfaces, dimension theory, the regular 17-gon, solution of equations by radicals, and the concept of infinity. The lectures are addressed to a lay audience. A mathematically untrained reader of the book (assuming that he has no difficulty with Tietze's picturesque German) can get a lot of understanding and