BOOK REVIEWS

Foundations of combinatorial topology. By L. S. Pontryagin. Trans. from the first (1947) Russian ed. by F. Bagemihl, H. Komm, and W. Seidel. Rochester, The Graylock Press, 1952. 12+99 pp. $3.00.

In the preface, the author states his purpose as follows:

"This book represents essentially a semester course in combinatorial topology which I have given several times at the Moscow National University. It contains a very rigorous but concise presentation of homology theory. The formal prerequisites are merely a few simple facts about functions of a real variable, matrices, and commutative groups. Actually, however, considerable mathematical maturity is required of the reader. An essential defect in the book is its complete omission of examples, which are so indispensable for clarifying the geometric context of combinatorial topology. In this sense a good complementary volume would be *Sketch of the fundamental notions of topology* by Alexandrov and Efremovitch,[1] in which the attention is focussed on geometric content rather than on the completeness and rigor of proofs. In spite of this shortcoming, it seems to me that the present work has certain advantages over existing voluminous treatises, especially in view of its brevity. It can be used as a reference for obtaining preliminary information required for participation in a serious seminar on combinatorial topology. It is convenient in preparing for an examination in a course, since the proofs are carried out with sufficient detail. For a more qualified reader, e.g., an aspiring mathematician, it can also serve as a source of basic information on combinatorial topology."

The reviewer considers that the author has done an excellent job of achieving his objectives within the limitations he has set down. Certainly none of the existing texts on combinatorial topology attains these objectives as well in so few pages. Experts may not always agree that he has chosen the clearest or most direct method of proof for some of his theorems, but of course this is largely a matter of taste.

The text consists of a two-page historical introduction, and three chapters. The introduction mentions the early ideas of Poincaré on homology theory, and the names of Alexander, Veblen, Lefschetz,

[1] Apparently this book has not been translated from the Russian. Such material may be found in Chap. VI of *Anschauliche Geometrie* by D. Hilbert and S. Cohn-Vossen or in *Einfachste Grundbegriffe der Topologie* by Paul Alexandroff.

Hopf, and Alexandrov in connection with the later development of the subject. In Chapter I the notion of a simplicial complex is introduced, and the homology groups of a finite simplicial complex are defined, using an arbitrary abelian group for coefficients. In Chapter II, it is proved that the homology groups are topological invariants, i.e. independent of the choice of a simplicial decomposition. The proof of invariance involves the use of simplicial mappings, the simplicial approximation theorem, and invariance under barycentric subdivision. In Chapter III, homology theory is applied to the study of continuous mappings and fixed points. It is proved that the homomorphism induced by a continuous map is invariant under homotopies. It is also proved that if the "Lefschetz number" of a mapping of a polyhedron into itself does not vanish, then the mapping has a fixed point.

There are a couple of results included which are digressions from the main line of development. In Chapter I it is proved that any n-dimensional compact metric space can be imbedded in Euclidean $(2n+1)$-space, and in Chapter II Sperner's Lemma is proved, and then used to demonstrate that the topological dimension of an n-simplex is actually n, and the Brouwer fixed point theorem.

Several topics which other authors might consider important are completely omitted from this small book. Examples of such topics are homology theory for general spaces (e.g., the singular or Čech homology theory), relative homology groups, cohomology theory, products, and duality theorems.

W. S. MASSEY

Bessel functions. Part II. *Functions of positive integer order.* Ed. by W. G. Bickley, L. J. Comrie, J. C. P. Miller, D. H. Sadler, and A. J. Thompson. (British Association for the Advancement of Science, Mathematical Tables, vol. 10.) Cambridge University Press, 1952. 40+255 pp. $11.00.

This is the tenth and final volume of the *British Association Mathematical Tables.* The Mathematical Tables Committee of the "B.A." has a long and honorable history; a brief account is included in the final report which has been reprinted in Mathematical Tables and Other Aids to Computation vol. 3 (1949) pp. 333–340. For many years this Committee represented probably the only organized effort to plan and compute in a systematic manner mathematical tables, and its work entailed cooperation between professional computers, mathematicians, and amateurs, between paid and voluntary workers. That this intricate system worked at all might be thought a minor