denoted by ***US***; the ***U***, presumably, stands for *Untermenge.*) Just as in the earlier edition, each section concludes with a useful collection of exercises.

A novel and laudable feature is the bibliography. This impressive list covers 137 pages; it is practically a separate publication in the same binding. (Possibly the publishers intend to make the bibliography and the main text available separately. The pages of the bibliography bear two numbers—one in continuation of the number system of the text and one independent of it.)

Readers not acquainted with *Einleitung* may gain an idea of the contents of the present book from the titles of its eleven sections; they are as follows. 1. Concept of set. Examples of sets. 2. The fundamental concepts. Finite and infinite. 3. Denumerable sets. 4. The Continuum. Transfinite cardinal numbers. 5. Ordering of cardinals. 6. Addition and multiplication of sets and cardinals. 7. Exponentiation of cardinals. The problem of infinitesimals. 8. Ordered sets. Similarity and order-types. 9. Linear sets of points. 10. General theory of well-ordered sets. Finite sets. 11. Ordinals and alephs. The well-ordering theorem.

PAUL R. HALMOS

Introduction to the theory of functions of a complex variable. By W. J. Thron. New York, Wiley, 1953. 10+230 pp. $6.50.

Here at last is a rigorous book on complex variables in the English language. There has long been a need for such a text and the author set for himself the task of filling this gap, and without doubt he has succeeded to a remarkable degree. The book is written in the Satz-Beweis style of Edmund Landau, each item being carefully labeled: Axiom, Theorem, Proof, Remark, etc., and in the reviewer's opinion this is of course the only possible style, where a rigorous exposition is the object in view. The book is divided into 31 short sections, instead of the more conventional longer chapters, and these shorter sections induce in the reader a comfortable feeling that not too much sustained effort will be required to read the book.

As every young instructor learns when he first strides into a class room resolved to make everything crystal clear to his pupils, the number system must first be put on a solid foundation, and this is not easy. Thron comes directly to grips with problem, and the allied problem of the foundations of geometry, and the first fourteen sections (more than one-third of the book) are devoted to these and related topics, and lie outside the domain proper of the theory of functions of a complex variable. These section headings are as follows:

1. Fundamental concepts, 2. Real numbers, 3. Cardinal numbers, 4. Complex numbers, 5. Sums and products, 6. Hausdorff spaces, 7. Metric spaces, 8. The plane of complex numbers, 9. Limits, continuity, differentiability, 10. Real functions of real variables, 11. Curves and regions in the plane of complex numbers, 12. Some combinatorial topology, 13. Jordan curves, 14. Rectifiable and directed curves. The material covered in these sections is just about what one would expect under these headings, and the only unusual item occurs in section 2. Here the author economizes on space and energy by advancing as an axiom the existence of a complete ordered field called the set of real numbers, and the reader is referred to Landau's Grundlagen for the proof that if there exists a set satisfying the Peano axioms then indeed there is a real number system.

Into the remaining space, devoted to complex variables proper, the author has packed a surprisingly large amount of material. In addition to all the standard topics one finds a proof of the Riemann mapping theorem, the construction of an elliptic modular function, and proofs of the Picard theorems. This large territory is covered by the simple device of leaving to the reader many of the easier proofs, and it is expressly stated in Remark 2.1 that if no proof is given for a theorem it means that the proof is left to the reader as an exercise. Despite this, the exercise lists at the end of the sections contain all too frequently the simple imperative "Prove theorem $X.Y$." In any revision of this text, all such redundant exercises should certainly be replaced by problems which lead the reader further into the subject.

The author seems to belong to the group of extreme purists who feel that a picture should never be used to clarify a situation. For most of the book, no great harm is done, since the average reader already has some idea of the appearance of a circle, curve, etc. However when the author decomposes the set of linear fractional transformations into elliptic, parabolic, hyperbolic, and loxodromic subsets, he certainly should have added in Remark 28.4 that the reader should see figures 6, 7, and 8 of Ford's *Automorphic functions* for a clear understanding of the content of Theorem 28.20. Better still, these illustrations could easily have been reproduced. Again, in the proof of the Cauchy integral theorem, it is the author's intention to decompose a directed triangle into four similarly directed triangles, but misprints actually give one similarly directed triangle, two inversely directed triangles, and one line segment. This misprint is easily caught by anyone already familiar with the Goursat proof, but the poor student who is approaching it for the first time receives the full punishment for the author's sin of omitting a figure here.

The book is relatively free of misprints, and the only other important ones which the reviewer found occur in the proof of the Riemann mapping theorem where the function $\sqrt{(g(z)-\alpha)/(g(z)-1)}$ should be replaced by $\sqrt{(g(z)-\alpha)/(\bar{\alpha}g(z)-1)}$, and the function $k(z)$ should be read $(h(z)-\sqrt{\alpha})/(\sqrt{\bar{\alpha}}h(z)-1)$.

More serious errors are also present. (1) In the proof of Painlevé's theorem, the strong form of Cauchy's theorem is used, but this latter has not been proved (a pardonable omission) nor is any mention made of this omission (an unpardonable one). (2) The definition of the Farey series of order n is given in a sophisticated manner by requiring in the definition that two successive terms p_1/q_1 and p_2/q_2 satisfy the condition $p_2q_1-p_1q_2=1$. This of course spares the author the trouble of proving this result as a theorem, but in the opinion of the reviewer is logically unsound, because it leaves open the question of whether there might be two or more Farey series of order n (with his definition this can occur). If all fractions $0\leqq p/q\leqq 1$, with $q\leqq n$, $(p, q)=1$, are to be included in the Farey series, this requirement assures us that there is at most one series of order n, but then this may turn out to be inconsistent with the condition $p_2q_1-p_1q_2=1$. (3) In Remark 23.3 the author states that his rather complicated definition of singular point is necessary "to avoid calling a point on a cut (other than a branch point) a singular point." As far as the reviewer could discover this is the only mention of the word cut in the entire book, and the remark only served to heighten the mystery, instead of elucidating it. (4) In the proof that on $|z|=r$, $\int z^{-1}dz=2\pi i$, the last assertion is unjustified, since no relationship has been developed between ϵ and η.

The author departs from standard usage in his definition of "conformal," and there seems to be no justification for this addition to the confusion in the meaning of words. On the other hand where a change should be made the author follows custom in defining a simple function in a region to be one that is single-valued holomorphic and takes no value more than once in the region. In the reviewer's opinion both the word simple and its international counterpart schlicht should be replaced by univalent for the simple reason that the latter is so suggestive of the more general concept of a multivalent function.

On the Jordan curve theorem, Thron states in his preface "The task I set myself demanded development of the basic tools of analysis and the inclusion of a proof of Jordan's curve theorem. . . ." On the other hand in a text appearing almost simultaneously Ahlfors states in his preface ". . . no proof is included of the Jordan closed curve theorem, which to the author's knowledge is never needed in function theory."

What is the place which Thron's book will occupy in the literature? Certainly it contains much valuable material, well organized and in convenient form for coordinated study. For this reason it belongs (a) in the library of every college which makes an attempt to teach mathematics, and (b) in the personal library of every specialist in function theory. However, just because it is so carefully written, with so much attention devoted to foundations, it should never (in the reviewer's opinion) be used as text either in a beginning or advanced course in function theory. The author is to be congratulated for his courage in writing such a book and his success in finding a publisher, and the publisher in turn has performed a real service for mathematics.

A. W. GOODMAN

Isoperimetric inequalities in mathematical physics. By G. Pólya and G. Szegö. (Annals of Mathematics Studies, no. 27.) Princeton University Press, 1951. 16+279 pp. $3.00.

The title of this book, as remarked by the authors in the preface (where the authors have admirably delineated the aims of the present work), suggests its connection with a classical subject of mathematical research, the "isoperimetric problem." This problem consists in seeking among all closed plane curves, without double points and having a given perimeter, the curve enclosing the largest area. The "isoperimetric theorem" gives the solution to the problem: of all curves with a given perimeter, the circle encloses the maximum area. If the perimeter of a curve is known, but the exact value of its enclosed area is not, the isoperimetric theorem yields a modicum of information about the area, an upper bound, an "isoperimetric inequality"; the area is not larger than the area of the circle with the given perimeter. There are, besides perimeter and area, many important geometrical and physical quantities (set functions, functionals) which depend upon the size and shape of a curve. There are many inequalities, similar to the isoperimetric inequality, which relate these quantities to each other. By extension, all these inequalities can be called "isoperimetric inequalities." Besides, there are analogous inequalities dealing with solids, pairs of curves (condenser, hollow beam), pairs of surfaces, and so forth. The present book is concerned with inequalities of this type.

An example of such an isoperimetric inequality, with which the subject matter of the book may be said to have begun, is the conjecture of B. de Saint-Venant (1856) concerning the torsion of elastic