BOOK REVIEWS

Darstellungstheorie unendlicher Gruppen und fastperiodische Funktionen. By W. Maak. Enzyklopädie der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Vol. I_1, No. 7, Part I. 2d ed. Leipzig, Teubner, 1953. 1–26 pp.

This is primarily a résumé of the theory of almost periodic functions on groups. Group representations are treated only in so far as they relate to almost periodic functions (for the most part), so that infinite-dimensional or unbounded representations get practically no discussion. The booklet is eclectic in style and moderately comprehensive though necessarily sketchy in detail.

More than half of it, the first chapter, is devoted to a general account of the theory of almost periodic functions on groups. The approach is essentially that of von Neumann and Weyl. Next there is a chapter on generalized almost periodic functions on groups, etc., necessarily a short chapter in view of the difficulty of treating such matters on general groups. As shown by Følner, the Weyl functions can to some extent be carried over from the real line, but the interesting Besicovitch functions have yet to be so extended, the key difficulty being that of handling discrete groups. Finally there is a chapter on miscellaneous aspects. It mentions among some other topics the Hilbert fifth problem for compact groups, the Freudenthal structure theorem for connected maximally almost periodic groups, the compact-discrete case of the Pontryagin duality theorem, and the original form of the Tannaka duality theorem.

In an encyclopedia article such as this one expects more comprehensiveness and up-to-dateness than in an independent introduction to the subject. Therefore one is disappointed to find that while it is a very readable account of the theory, it deals mainly with the theory as it existed a few years ago and has some notable omissions. To be more specific: (1) little mention is made of recent pertinent topological and algebraic developments. For example, it is stated that it has not yet been possible to set up the basic theory without the use of the theory of integral equations. While this may be true in spirit, it may well be misleading to have no indication whatsoever of how Banach algebras or partially ordered linear spaces can be used effectively to treat almost periodic functions. (2) Although the author discusses spherical harmonics and the decomposition of the regular representation of a compact group, the essentially equal simple structure of an arbitrary representation of a compact group is not described.

(3) No mention is made of Chevalley's version and application of the Tannaka theorem, nor of the work of Krein in related connections. (4) It is mentioned that the unimodular group in $n(>1)$ dimensions is minimally almost periodic; notably more generally, the same is true for any noncompact simple Lie group.

There is some occasional vagueness that may bother the non-expert reader. Thus, in the discussion of Hilbert's fifth problem, "finite-dimensional" is used as if it meant "finite-dimensional and locally euclidean," or at least "finite-dimensional and locally connected." And in quoting van der Waerden's result on the continuity of representations of semi-simple groups, the author neglects to make explicit the qualification that the representation be finite-dimensional.

This booklet conveys the scope of the theory of almost periodic functions on groups more rapidly and pleasantly than any other treatment with which the reviewer is familiar, and as such performs a valuable and important service.

I. E. SEGAL

Undecidable theories. By A. Tarski in collaboration with A. Mostowski and R. M. Robinson. Amsterdam, North Holland Publishing Company, 1953. 9+98 pp. $2.50.

The material of this book is based on results which were obtained by the authors, in particular by Tarski, between 1938–1952, and were originally intended to be published as a series of papers in some mathematical journal. Fortunately they are combined here in one volume.

A given axiomatic theory T is called decidable or undecidable according as we can or cannot find a mechanical procedure which permits us to decide (in a finite number of steps), for each particular sentence formulated in the symbolism of T, whether this sentence is provable by means of the devices available in T. (The number of steps necessary will, however, in general depend on the structure of the sentence under test, just as in applications of the Euclidean algorithm the number of divisions necessary depends on the two numbers whose g.c.d. we wish to find.) The decision problem of a theory T is the problem of determining whether T is decidable or undecidable. The problem of deciding which axiomatic theories have decision procedures and which do not is one of the central problems of modern symbolic logic, and should be of interest to all mathematicians, not just logicians. Here the authors describe all the principal methods and results in this field. The exposition throughout is