

price and handsome physical appearance. They are a "must" on the analyst's bookshelf.

PAUL T. BATEMAN

*Methods of mathematical physics.* By R. Courant and D. Hilbert. Vol. I. 1st English ed. New York, Interscience, 1953. 16+561 pp. \$9.50.

This book is mainly a translation of the second German edition of the celebrated textbook written by Courant and his collaborators at the University of Göttingen. While Hilbert did not take any active part in its preparation his name was put as a co-author to indicate the tremendous influence which he exerted on the mathematical thinking of his surroundings and indeed of the whole mathematical world. Since the content of the book is well known to every worker in the field, let us recall only that the text covers the following subjects: linear transformations and quadratic forms, development of arbitrary functions in series of orthogonal functions, linear integral equations, calculus of variations, eigenvalue and vibration problems, application of variational calculus to eigenvalue problems, and special functions.

This work presents a cross-section of the subject matter as it appeared to Courant's school in Göttingen in 1931. It is, of course, not a valid criticism that the translation contains no newer developments, inasmuch as the author states in his preface that the pressure for publication of an English "Courant-Hilbert" became irresistible. However, regardless of its contents as seen today, one may reasonably ask what will be the reaction of some students who will miss the possibility of using the work as a reference book in which each theorem is stated and numbered in a precise way. Instead of a catalogue of theorems, the reader will find an artistic exposition of the profound meaning of mathematical thinking. The author is greatly aided in his exposition by his natural inclination to somewhat fluid statements which greatly stimulate the imagination of the reader. The reviewer gratefully acknowledges being one of the large community of scientists outside of Göttingen who were influenced by Courant's book.

Very few additions and alterations have been included in the English edition; of these, only the interesting appendix by W. Magnus has been mentioned in the preface. This appendix deals with the question of how a set of linearly independent spherical harmonics in three variables is transformed if the coordinate system is rotated.

The main addition not mentioned in the introduction is a paragraph entitled: Reciprocal quadratic variational problems (chapter 4, §11, pp. 252-257), which complements a preceding section (§9) de-

voted to the discussion of upper and lower bounds of quadratic functionals and to Friedrichs' analysis of Trefftz's method. In the new addition, the author emphasizes a geometrical approach given by Prager and Synge (*Quarterly of Applied Mathematics* vol. 5 (1947) pp. 241-269). Similar questions were widely discussed by various methods in recent publications, and a part of the literature is mentioned by Courant in one of the longest footnotes of the book. In view of the great and deserved influence of the book, such a footnote cannot be passed over without comment. The footnote states that the theories indicated in §9 have been recently rediscovered and advanced by several authors. Let us emphasize that the word rediscovered is incorrect, since all papers refer either to Trefftz's method or to its analysis by Friedrichs. The purpose of the recent investigations was to simplify the method of Trefftz and to obtain stronger results. Incidentally, the footnote omits a reference to the work of Diaz and Weinstein (see e.g. *Schwarz' inequality and the methods of Rayleigh-Ritz and Trefftz*, *Journal of Mathematics and Physics* vol. 26 (1947) pp. 133-136) which uses an analytic approach. The various methods were recently analyzed by Diaz, *Collectanea Mathematica* vol. 4 (1951) pp. 1-47, specially pp. 41-46, who pointed out the advantages of the analytic method, which yields simpler formulas and more correct results than the geometric procedure.

The tendency of understatement of the work done outside the author's circle reappears in the footnote on page 175, where Lord Rayleigh's contributions to variational methods are evaluated as follows: "Even before Ritz, such ideas were successfully employed by Lord Rayleigh."

The book preserves not only all the high points of the original, but also some of its misprints, which, being by now classical, only add to the pleasure of the reader. It is with great expectations that the reviewer is looking forward to the translation and additions to the second volume.

A. WEINSTEIN

*Forced oscillations in non-linear systems.* By C. Hayashi. Osaka, Nippon Printing and Publishing Co., 1953. 14+164 pp. \$4.50.

This book was written "to provide engineers and physicists with practical knowledge concerning the important subject of non-linear oscillations," in particular forced oscillations governed by non-linear equations of the second order.

The text is divided into two parts; the first concerns the stability of steady state oscillations, whereas the second is devoted to a dis-