cussion of the geometry of solutions of a slightly perturbed system of two first order equations. The first chapter considers the equation $v''+f(v)v'+g(v)=e(t)$ $('=d/dt)$, where e is periodic. The existence of a periodic solution of the same (or integral multiple of the) period of e is assumed, and the stability of this solution is defined in terms of the characteristic exponents of the corresponding equation of the first variation. A discussion of the Mathieu equation and Hill's equation follows in which "approximate" characteristic exponents are obtained, and corresponding stability regions are plotted. The question of the degree of approximation is not considered. In the next three chapters special cases are worked out in detail, a typical equation being $v''+2\delta v'+(c_1v+c_3v^3)=B\cos\nu t$, where δ, c_1, c_3, B, ν, are constants, with a subharmonic of the form $v=k_1\sin t+k_2\cos t+k_3\cos\nu t$ assumed. Various experiments with electrical oscillatory circuits are described and the results compared with the approximate stability regions determined. In the last two chapters (Part II) the integral curves of some special cases of the system $x'=X(x, y)$, $y'=Y(x, y)$ are depicted in the vicinity of the equilibrium points. The solutions of the van der Pol equation are also sketched.

The book is replete with excellent illustrations.

Although from the mathematical viewpoint the equations and solutions considered are rather special, this work should serve its purpose well, and in addition provide mathematicians with a supply of nicely illustrated examples of forced oscillations.

E. A. CODDINGTON

Hypergeometric and Legendre functions with applications to integral equations of potential theory. By Chester Snow. (National Bureau of Standards, Applied Mathematics Series, no. 19.) Washington, Government Printing Office, 1952. 11+427 pp. $3.25.

This is the second edition of a monograph of which the first edition appeared in 1942 (as MT15 in the Mathematical Tables Series of the National Bureau of Standards) and was exhausted within a year, a second printing being sold out soon afterwards. Continuing, and indeed increasing, demand for the monograph and the impending retirement of the author from active service with the National Bureau of Standards were the motivations for the present second edition. Known misprints have been corrected, some chapters have been rewritten and expanded, and a new chapter (on confluent hypergeometric functions) has been added.

A brief indication of the contents, chapter by chapter, follows.

I. Definitions and preliminary formulas relating to the gamma function and Gauss' hypergeometric series.

II. Homographic substitutions (linear transformations) of the hypergeometric differential equation.

III. Non-linear substitutions (quadratic transformations) of the hypergeometric differential equation.

IV. Integral representations of the hypergeometric function.

V. A few relations of contiguity.

VI. Associated Legendre functions. It should be noted that in this book $P_\nu^\mu(z)$ and $Q_\nu^\mu(z)$ stand for functions which are numerical multiples of the associated Legendre functions defined by Hobson, and commonly used, in the z-plane cut along the real axis from $-\infty$ to 1. The functions defined in the complex plane cut from $-\infty$ to -1 and again from 1 to ∞ are denoted by $T_\nu^\mu(z)$ and $q_\nu^\mu(z)$, and the first of these differs again by a constant factor from MacRobert's $T^{-\mu}_\nu(z)$.

VII. Heun's function, which is a standard solution of an ordinary linear differential equation of the second order with four regular singular points and prescribed exponents at these singular points.

VIII. Some integral representations. This is one of the longest chapters of the book, it is one of its most original portions, and has been expanded considerably in the new edition. The general problem is the representation of "arbitrary" functions as integrals involving solutions of a given differential equation (or rather of a differential boundary value problem with a continuous spectrum), in a sense the problem of the generalized Fourier integral. After a general formulation of the problem, integral representations in terms of hypergeometric, Legendre, Bessel, and related functions are considered and illustrated by a considerable number of interesting examples.

IX. Some integral equations of potential theory with $Q_{m-1/2}$ as nucleus; canonical expansions of $Q_{m-1/2}$. Another long and highly original chapter. The problem of determining a potential whose only sources are simple layers on surfaces of revolution leads to integral equations of the first kind whose nuclei are Legendre functions, m being an integer. Such integral equations are discussed in various systems of curvilinear coordinates in the meridian planes, and the bilinear expansions of the nuclei in series of characteristic functions of the integral equations are the canonical expansions of $Q_{m-1/2}$. Of particular importance are those curvilinear coordinate systems in which Laplace's equation is separable, and a determination of all such coordinate systems is given in this chapter.

X. Applications. The results of the last two chapters are used to construct potentials determined by certain data on given surfaces of revolution. The answers are given in (circular) cylindrical, (spherical) polar, dipolar, toroidal, prolate and oblate spheroidal, paraboloidal, and annular (cyclidical) coordinates. This is the longest chapter of the book, and is full of useful information. The thoroughness in laying the groundwork in chapters VIII and IX simplifies the work of this chapter considerably.

XI. Integral representations with integrals of confluent hypergeometric functions. This chapter has been added in the second edition.

As J. H. Curtiss remarks in the Foreword, this work is "a labor of love on the part of the author" with whom "to derive new formulas pertinent to the hypergeometric function was, quite literally, his hobby as well as his profession." The resulting book has a flavor all of its own which sets it apart from all other books on the subject. The monograph is written by a specialist for mathematicians seeking highly specialized information; it does not attempt to replace, or to compete with, standard texts, and offers much that will be new even to experts in this field. The continuing demand for the book is due in a large measure to the increasing number of mathematicians who have "discovered" Snow's monograph and found it so helpful that they would like to own a copy. In the course of the last ten years the present reviewer loaned his copy of the first edition to numerous colleagues, and on the book being returned (somewhat reluctantly in many cases), the borrower almost invariably asked how he could buy a copy. It is a pleasant thought that in the future it will be possible to give a simple answer to this question.

A. ERDÉLYI

Introduction to the theory of stochastic processes depending on a continuous parameter. By H. B. Mann. (National Bureau of Standards Applied Mathematics Series, no. 24.) Washington, Government Printing Office, 1953. 45 pp. 30 cents.

To have written a serious textbook on the theory of stochastic processes in the small compass of forty-five pages is an astonishing *tour de force* and the reviewer is full of admiration for the ingenuity with which so much has been packed into so small a space. But brevity can be an enemy of clarity and even the "educated mathematician" to whom (in the foreword by Dr. J. H. Curtiss) the argument is addressed may find some difficulty in discovering from the evidence presented what a stochastic process *is* (unless he already knows). The following notes, supplementing Chapter I, may help in pinpointing the author's point of view in relation to other surveys.

Professor Mann starts by defining an (indexed) family of random variables $\{x_t: t \in T\}$, and he calls it a stochastic process if the index-set T is a set of real numbers. If we ignore this distinction (and it is