There is a slip in the transfinite induction theorem. It seems to permit one to prove a proposition true for "all ordinals." Incidentally, the Burali-Forti paradox on the set of all ordinals is explained two pages before this theorem. Aside from this, the reviewer found only typographical errors—and not very many of these. The format of the book is quite pleasing.

M. E. MUNROE

Differential line geometry. By V. Hlavatý. Trans. by H. Levy. Groningen, Noordhoff, 1953. 6+10+495 pp. 22.50 Dutch florins; cloth, 25 Dutch florins.

This book was first published in Czech in 1941 and was later translated into German and published by Noordhoff in 1945. The present edition is a translation from the German edition with the author's collaboration. The translator himself has added to the text by suggesting changes in some theorems and by adding a few new ones.

The book is meant to be a definitive work on three-dimensional differential line geometry, where line geometry in 3-space is studied as point geometry on a 4-dimensional quadric in a projective point space of 5 dimensions. Klein discovered the mapping of line geometry onto the 4-dimensional quadric, and for this reason the quadric is called the Klein quadric (or K-quadric) and the 5-dimensional space the Klein- (or K-) space. This viewpoint of three-dimensional line geometry has been used by authors before, but never as extensively as in this text. Both the classical material on the subject and new contributions by the author have been included.

The text is necessarily quite long and the author has tried to overcome some of the difficulties of length by dividing his work into "books," each of which can be read without the others. This naturally leads to some repetition of material. The books are divided into chapters which are numbered consecutively throughout the text. There are five books: the first (Chapter I) an introduction to line space; the second (Chapter II) on ruled surfaces; the third (Chapters III, IV, and V) on congruences; the fourth (Chapters VI–IX) on complexes, and the last (Chapter X) on line-space. Tensor calculus is used at all times, and simplifies the notation. For those readers who are unfamiliar with the tensor calculus the author has included in an appendix a straightforward, well written account of that part of the subject necessary for a reading of the text.

In the first book the author defines Plücker coordinates, Klein points, and Klein 5-dimensional space. He states that all topics, as far as possible, will be discussed from the viewpoints of projective, affine, and metric geometry. Unless explicitly stated otherwise, all theorems and definitions are given for projective geometry.

The standard material on ruled surfaces in projective differential geometry is given in the second book, ruled surfaces appearing in K-space as K-curves on the K-quadric. Also five particular linear complexes are introduced, as are three projective curvatures which determine a K-curve (hence the corresponding ruled surface) uniquely to within initial conditions. By use of one of the five special complexes, many of the familiar metric properties of ruled surfaces are derived.

More or less well known material on congruences, which are represented by K-surfaces in K-space, is presented in Chapter III. Chapter IV deals with congruences in affine and metric geometry, from both the algebraic and analytic viewpoints. Two uniquely defined metric tensors are introduced and used in the study of the congruences. Scalars representing the first and second mean curvatures of a congruence are defined in terms of these tensors. Normal congruences are studied, as are principal surfaces of congruences, distribution curvature, central points, and central planes. Pseudo-parallelism, autoparallelism, and teleparallelism are discussed and Frenet equations are developed for surfaces on congruences.

In Chapter V the relations between the K-surface and K-space are studied. For example, the fundamental equations of Weingarten, of Mainardi-Codazzi, and of Gauss are derived for congruences whose second osculating K-spaces are 5-dimensional (4-dimensional, 3-dimensional).

The complexes studied in the fourth book are not necessarily linear. Chapter VI deals with foundations and with fundamental equations concerning 3-dimensional manifolds in K-space (the images in K-space of complexes in 3-space). Algebraic complexes are studied, as well as the nature of a complex in the neighborhood of its singular and special lines. The fundamental equations are derived.

Chapter VII is concerned with surfaces of a complex (C-surfaces). Elemental C-surfaces and elemental C-congruences are discussed, and a normal curvature for nondevelopable C-surfaces is defined. Definitions are given for two C-surfaces to be autoparallel, and for two nondevelopable C-surfaces to be extremal surfaces. It is shown that a necessary and sufficient condition that two nondevelopable C-surfaces be autoparallel is that they be extremals. A non-extremal surface has two curvatures which satisfy generalized Frenet equations and which determine the surface uniquely to within initial conditions.

Absolute pseudoparallelism, absolute autoparallelism, and absolute teleparallelism are discussed briefly.

Under the mapping onto the K-quadric a congruence of a complex is represented by a K-surface contained in the 3-dimensional K-manifold which is the image of the complex. A normal surface of a congruence is defined in Chapter VIII, and it is shown that this surface is projectively orthogonal to every surface through the same line as the normal surface. Generalized fundamental equations are derived and used in determining whether there exists a C-congruence with two particular preassigned tensors.

Chapter IX deals with surfaces of a congruence of a complex (called CC-surfaces). Elemental principal CC-surfaces, characterized by extreme values of the outer curvature, are studied. Spheroidal lines, which are lines on which the elemental principal CC-surfaces are not determined uniquely, are discussed. Inner and outer curvatures of a CC-surface are defined. The inner curvature determines the CC-surface in the congruence to within initial conditions. These curvatures are closely related to the first curvature of the surface considered as a surface of the complex. The concept of torsion of a CC-surface leads to an equation connecting the invariants of the CC-surface. Brief mention is made of asymptotic surfaces of C-congruences.

The last book is devoted to a study of the entire line space represented by the K-quadric. This space is shown to be conformal-euclidean. A section is devoted to metric line space and there it is shown that the curvature tensor can be made to vanish. A short section is devoted to the resulting analagmatic geometry.

There are naturally advantages and disadvantages to the exclusive use of the mapping of line-space onto the K-quadric. Questions concerning line manifolds can be answered fully but, as the author himself points out, questions of point geometry must be left unanswered.

The exposition is clear throughout. Although this book has been written as a textbook, it is highly specialized and will probably be used more as a reference than as a text. The author has placed problems in the text, not at the end of sections, but where they arise naturally in the theory. The proofs of some of the simpler theorems have been omitted and left as exercises.

There is no bibliography, and this is a serious defect. Only a few references to other works are given. There is too much "organization" in the writing. For example, there are Remark I, Agreement (1,1), Theorem (1, 1), Definition (1, 1), etc.

There are a few minor misprints, but on the whole the book has been very well printed and proofread—this latter not always being the case today. The book is a valuable addition to the literature on line geometry, and a good translation into English makes it available to many more readers.

ALICE T. SCHAFER

Drei Perlen der Zahlentheorie. By A. J. Chintschin. Trans. from the 2d (1948) Russian ed. by W. v. Klemm. Berlin, Akademie-Verlag, 1951. 61 pp. 6.50 DM.

Three pearls of number theory. By A. Y. Khinchin. Trans. from the 2d (1948) Russian ed. by F. Bagemihl, H. Komm, and W. Seidel. Rochester, Graylock, 1952. 64 pp. $2.00.

The author, one of the leading Russian mathematicians, attempts to present three important recent results in such a way that they can be understood without much knowledge of number theory. He tries to create admirers of number theory by showing that elementary number theory is not yet a finished field since highly interesting new results were obtained by ingenious methods during the last few years, and further progress can be expected.

The author has been extremely successful in writing an excellent book for trained mathematicians. However, it is stated in the German edition that the book can be read by students of the upper grades of high schools and amateurs of mathematics and in the American edition that it can be understood by beginning college students. It is the reviewer's opinion that this is impossible. No such reader could study it with success. Even if he could understand some pages, he would not recognize the beauty of the results and their proofs.

The simplest part of the book is certainly the first chapter. The reviewer has proved its results in his classes at the University of Berlin and at the University of North Carolina, and he knows from this experience that it is not easy to present these theorems even to students who had taken a course in number theory.

In the first chapter the author proves the following theorem of van der Waerden published under the title *Beweis einer Baudetschen Vermutung*, Nieuw Archief voor Wiskunde (2) vol. 15 (1927) pp. 212–216. Let k and l be arbitrary integers. There exists a constant $W=W(k, l)$ such that for any distribution of the numbers $1, 2, \cdots, W$ into k classes at least one of the classes contains an arithmetic progression of l terms.