

unpublished results having been included. The bibliography, while not complete, is sufficiently extensive to be quite useful. At present, it is the only single work which could reasonably serve as an introduction to both linear and nonlinear elasticity. It is perhaps too specialized to be well suited for this purpose, though a comprehensive treatment would fill several volumes.

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Ricci-Calculus. An introduction to tensor analysis and its geometrical applications. By J. A. Schouten. 2d ed. Berlin, Springer, 1954. 20+516 pp. 55 DM; clothbound, 58.60 DM.

Since the publication in 1901 of the famous paper on absolute differential calculus by G. Ricci and T. Levi-Civita which established the foundation of the so-called Ricci-Calculus and especially since the publication in 1916 of the theory of general relativity by A. Einstein, the importance of Ricci-Calculus in its geometrical and physical applications has been universally recognized. The first edition of this book, published in German in 1923, covered all the researches made until then and played an important instructive role in this new branch of geometry. But since then tensor calculus was further developed to a great extent and many new notions were introduced, for example, normal coordinates, the symbolism of exterior differential forms, infinitesimal deformations, Lie derivatives, subprojective spaces and their generalizations in Hermitian geometry. To cover these new notions, J. A. Schouten and D. J. Struik published in 1935 and 1938 their *Einführung in die neueren Methoden der Differentialgeometrie* I and II. In this book they introduced the so-called kernel-index method which is a characteristic of Schouten's school.

Since 1935 Ricci-Calculus was again further developed. For example, the projective and conformal geometries have been studied in great detail from various points of view; Finsler and Cartan spaces, general spaces of paths and those of K -spreads were introduced, the motions in these spaces were studied by the use of Lie derivatives; and the ideas of harmonic spaces and of spaces of recurrent curvature were developed by British mathematicians. The book under review was written to cover these new developments in the Ricci-Calculus and the author tries to retain the instructive and the encyclopedic character which "Der Ricci-Kalkül" has. Thus, although the book is entitled "Ricci-Calculus, the second edition," it is an entirely new book.

It consists of eight chapters. The first chapter is devoted to tensor algebra. In the first section, an n -dimensional affine space E_n is

defined and the coordinate transformations and point transformations are discussed. In this section the principle of the kernel-index method is explained. We use the same kernel letter to indicate a geometric object and different kinds of indices to indicate its components with respect to different kinds of coordinates. So, with coordinate transformations the kernel letters do not change but a new set of running and fixed indices is introduced. But with point transformations the kernel letters change and the running and fixed indices remain the same.

In the following sections, the author defines and discusses successively quantities in E_n , algebraic operations applied to quantities, subspaces in E_n , rank of a quantity with respect to one or more indices, symmetric tensors, multivectors, tensors of valence 2, introduction of a metric in an E_n , hybrid quantities, and abridged notations. In the last section, the author gives a survey of many different notations. At the end of Chapter I, we read: "Young authors especially, who cannot yet foresee all the consequences, should abstain as far as possible from introducing ad hoc abbreviations."

Chapter II is devoted to analytic preliminaries. The author starts by defining the arithmetic n -dimensional space \mathfrak{A}_n and the geometrical n -dimensional space X_n . The development is similar to that of O. Veblen and J. H. C. Whitehead. Then he discusses geometric objects in X_n , the X_m in X_n , the (non-holonomic) X_n^m in X_n , operators Rot and Div, Pfaff's problem, the theorem of Stokes, anholonomic coordinates, the Lie derivatives and the Lagrange derivatives. In the last section, he compares his own symbolism with that of E. Cartan. This section should be useful for readers who are already familiar with notations other than those of the present book.

In Chapter III, the author comes to the discussion of linear connexions. After preliminary discussions in E_n , he defines the parallel displacements in X_n in a quite general way. Then there follow discussions on the torsion tensor, the curvature tensor, identities satisfied by the curvature tensor, integrability conditions of some tensor equations, geodesics and normal coordinates, Fermi coordinates and formulas in anholonomic coordinates. At the end of this chapter, the author again compares his formulas with those of E. Cartan. The last section is devoted to the linear connexions depending on a non-symmetric fundamental tensor, which A. Einstein used in 1945 to obtain a unified field theory.

Chapter IV is devoted to Lie groups and linear connexions. The classical theory of Lie groups is developed by an ingenious use of the Ricci-Calculus. This chapter deals with finite continuous groups,

the parameter groups and the adjoint groups of a finite continuous group, finite continuous transformation groups, the geometry of group space, invariants of a transformation group in the X_n , of the ξ^* , invariants of a group in group space, and properties of integrable groups. Unfortunately the theory of simple and semi-simple groups, developed by E. Cartan and the author himself, is excluded.

In Chapter V, the author discusses imbedding and curvature. He starts with curves and hypersurfaces in a Riemannian space and in a space with a linear connexion. He next develops the theory of general (holonomic and anholonomic) subspaces making use of the D -symbolism due to van der Waerden and Bortolotti. The chapter ends with discussions on product spaces.

Chapter VI is devoted to projective and conformal transformations of connexions. The author first discusses projective transformations in a space with a symmetric connexion and those in a Riemannian space. He next deals with the behavior of affine conics under a projective transformation. In §4, the projective connexions of T. Y. Thomas are discussed but without using one supernumerary coordinate. In §§5 and 6, the author discusses the conformal transformations in a general Riemannian space and also in an Einstein space. Next comes the theory of conformal connexions of J. M. Thomas. The rest of the chapter is devoted to the subprojective connexions of Kagan, Adati's problem on subprojective spaces, subprojective transformations of a linear connexion, and concircular transformations in a Riemannian space.

In Chapter VII, variations and deformations are discussed. The author first explains the general method for dealing with the deformation problems and gets some of the classical results on motions and affine motions in a Riemannian space and in a space with a linear connexion, as well as some recent results by I. P. Egorov, G. Vranceanu, and the reviewer. He then comes to the discussions of deformations of subspaces. The latter part of the chapter is devoted to the discussions of holonomy groups and the symmetric space of E. Cartan. In the last section, the author applies the method of moving frames of E. Cartan to the study of holonomy groups and of symmetric spaces using his own symbolism. It will be very useful to see the relation between the symbolism of E. Cartan and that of the author.

The last chapter entitled "Miscellaneous examples" contains discussions of harmonic Riemannian spaces, connexions in complex spaces, and spaces of recurrent curvature.

The book has an extensive bibliography. It contains 1400 papers

by 350 authors. The author says that in order to retain something of the encyclopedic character of the first edition, he selected the titles in such a way that the reader interested in some topic will always find at least a few titles which can lead him on to more references. The bibliography indicates where each paper is quoted in the text, which is very convenient for the reader.

The main topics in differential geometry which are not treated in this book are the geometries of Finsler and Cartan spaces and their generalizations, the geometries of paths and K -spreads and their generalizations, projective and conformal geometries with the use of supernumerary coordinates, the geometry under contact transformations, the geometry in almost complex spaces, the theory of general geometric objects, the differential geometry of fibre bundles, etc. The reviewer sincerely hopes that the author may soon have another opportunity to give a survey of these interesting topics.

This excellent book by an author who has since 1918 always been a pioneer of research in the fields of differential geometry will serve not only as "an introduction to tensor analysis and its geometrical applications" but also as an encyclopedia for the differential geometers of the front line and it will give all the information in the small necessary for the development of differential geometry in the large.

KENTARO YANO

Lectures on partial differential equations. By I. G. Petrovsky. Trans. by A. Shenitzer. New York, Interscience, 1954. 10+245 pp. \$5.75.

This book is intended to be an introductory and self-contained text in the theory of partial differential equations. That it fulfills this purpose is a result not only of the excellent presentation of the author but also of the excellent translation of A. Shenitzer which preserves the easy, flowing style of the original.

The book is divided into four chapters: the classical trichotomy, hyperbolic, elliptic and parabolic equations, and an introductory chapter devoted to motivation and notation.

The first chapter begins with a derivation of the various classes of equations mentioned above from prototype physical problems. Beginning the study of existence and uniqueness, the author presents the Cauchy-Kowalewski theorem for the Cauchy problem with analytic initial conditions. This leads to the concept of characteristics, and to the question of existence and uniqueness of solutions with nonanalytic initial conditions. There is a very thorough discussion of this topic, in which the author brings the reader up to date on some of the latest developments in the field. It is characteristic of the book