Danwakai, 240 (1942). P. 114, line 13: read "J. Reine Angew. Math., 196" for "Math. Zeitschrift."

To summarize: in the reviewer's opinion, this is an important and well-written book which should help to stimulate research on the classical groups. The book not only gives a thorough exposition of the present state of the subject, but is also an excellent introduction to the modern techniques basic to further work in this field.

IRVING REINER

Relativity: The special theory. By J. L. Synge. New York, Interscience, 1956. 7+415+20 pp. $10.50.

In the preface of this book Synge states, "The basic idea of this book is to present the essentials of relativity from the Minkowskian point of view, that is, in terms of the geometry of space-time." This reviewer agrees that an exposition of the special theory of relativity based on such an idea is sorely needed and the author's "Ambition . . . to make space-time a real workshop for physicists, and not a museum visited occasionally with a feeling of awe" is laudable.

On the whole this is a well-written discussion of the following topics in special relativity: Kinematics, mechanics of single particles and systems of particles, mechanics of a continuum and electromagnetic theory. These topics are covered with varying degrees of thoroughness, completeness and quality of exposition.

The first chapter discusses the relationship between the metric of space-time and physical measurements, the latter being described in terms of ideal experiments. The author's intention is to lay a foundation strong enough to support both the special and general theories. This intention is fulfilled in a lively thought-provoking way.

The next four chapters are devoted to the geometry of flat space-time (Minkowski space), the group of this space (the Lorentz group), and the explanation of the classical experiments which were first satisfactorily accounted for by the Einstein special theory of relativity. The discussion of these topics is clarified greatly by using space-time diagrams in an effective manner.

Chapter IV which deals with the proper homogeneous Lorentz transformations which do not interchange the past and the future contains as one of its main theorems the incorrect statement: "Any finite Lorentz transformation (of the restricted class defined (above)) is equivalent to a 4-screw." A 4-screw is defined as "a rotation in a time-like 2-flat π, followed (or preceded by . . .) a rotation in a space-like 2-flat π^*, the 2-flats π and π^* being orthogonal to one another." The matrix representing a 4-screw may be taken to be

$$\begin{Vmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cosh x & \sinh x \\ 0 & 0 & \sinh x & \cosh x \end{Vmatrix}.$$

In this case the 2-flat π is the x^3, x^4 plane and the 2-flat π^* is the x^1, x^2 plane.

The parabolic Lorentz transformation which in real coordinates ($x_1=x$, $x_2=y$, $x_3=z$, $x_4=ct$) has the form

$$L = \begin{Vmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1/2 & 1/2 \\ 1 & 0 & -1/2 & 3/2 \end{Vmatrix}$$

furnishes a counterexample to the theorem quoted. For, its only proper values are $+1$ and its elementary divisors are not simple. In fact if P, U and V are the vectors given by

$$P^\sigma = \delta_3^\sigma + \delta_4^\sigma, \qquad V^\sigma = \delta_2^\sigma, \quad \text{and} \quad U^\sigma = \delta_1^\sigma$$

then

$$LP = P,$$
$$LV = V,$$
$$LU = U + P.$$

The only null-vector left invariant by L is P. The three-space tangent to the light cone and spanned by the vectors P, U and V is left invariant under the transformation L and the two-space spanned by P and V is left pointwise invariant.

The author's discussion does not properly take into account the possibility of the two real null-vectors left invariant by the 4-screw coinciding. The theory of such parabolic Lorentz transformations can be given quite readily in terms of the two component theory of spinors which the author discusses too briefly for the reviewer's taste.

In Chapters VI and VII which deal with the mechanics of a particle and the mechanics of a discrete system the laws of conservation of energy and momentum, that is, the conservation of the momentum four vector, are treated in detail for a variety of problems including problems involving the interaction of material particles and photons. The notion of angular momentum is introduced and discussed by means of an antisymmetric second order tensor.

Chapter VIII deals with the mechanics of a continuum, the latter being considered as a limit of a discrete system described by a distribution function which in turn specifies the number of particles of various types being considered. Properties of the distribution functions are not considered; instead attention is directed to various average quantities. In the first part of the chapter density, pressure and temperature of a gas are defined and discussed. In the second part of the chapter the stress energy tensor of a continuum of particles interacting only via collisions is derived. For the case of the perfect fluid the time-like proper value of this tensor is said to be density and the three coincident space-like ones are said to be pressure. The relation between these definitions and those given at the first part of the chapter is not discussed. In particular it is not mentioned that the time-like proper value is not in general the proper density of proper mass as defined in the earlier sections but is energy density of the continuum as measured by an observer whose time axis is in the direction of the time-like proper vector λ_r associated with this proper value. In general the energy density associated with proper mass will be different from the energy density given by the time-like proper value. This difference has been called the internal energy density of the fluid described by the stress energy tensor and is the relativistic analogue of an important concept in classical hydrodynamics.

In discussing relativistic hydrodynamics the author deals only with four conservation laws, those contained in the statement that the divergence of the stress energy tensor vanishes. He does not remark on the fact that classical hydrodynamics of a single fluid involves five conservation laws: the conservation of energy, momentum and mass. The first four are treated relativistically by means of the stress energy tensor. The fifth is contained in the statement that the divergence of the vector, obtained by multiplying the proper density of proper mass by the mean velocity vector, vanishes.

The final two chapters of the book, Chapters IX and X are devoted to electromagnetic theory. In the first of these entitled the *Electromagnetic field in vacuo*, considerable space is devoted to the geometry of second order antisymmetric tensors and to that associated with the symmetric stress energy tensor associated with the antisymmetric tensor describing a Maxwell field. It is regrettable that the author does not point out that when the determinant of the stress energy tensor is nonvanishing, the coefficients of this tensor are proportional to those of a proper-Lorentz matrix of period two. Thus, the extensive geometrical discussions of Chapter IX have an important bearing on Chapter IV.

Chapter IX contains a discussion of solutions of the scalar wave equation which vanish at infinity and have no singularities. Solutions of the Maxwell field equations are generated from these. It is shown that the space integral of the stress energy tensor for these solutions can be made to be proportional to a time-like vector with a finite proportionality factor. It is suggested that these solutions correspond to particles. Discontinuous electromagnetic fields called electromagnetic shock waves are also discussed.

Chapter X deals with electromagnetic fields due to moving singularities and also with Maxwell's equations in moving matter. The author uses very effectively the four dimensional formulation and geometric properties of Minkowski space to give a concise and clear treatment of a variety of topics on the behavior of fields and charges.

It is hoped that the faults pointed out above will be corrected in some future edition of the book. If this is done the book will be a valuable addition to the literature for it has many virtues and the basic idea behind it is a very sound one.

A. H. TAUB

BRIEF MENTION

Convegno internazionale sulle equazioni lineari alle derivate parziali. Trieste, 25–28 Agosto 1954. Ed. by the Unione Matematica Italiana with the assistance of the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1955. 13+233 pp. 3000 lire.

This volume contains, besides an introduction by Sansone, papers by Agmon, Bers and Nirenberg, Cimmino, Courant, Diaz, Fichera, Miranda, Pleijel, Synge, Tautz, Tricomi, and Weinstein.

Functional analysis. By F. Riesz and B. Sz.-Nagy. Trans. by L. F. Boron from the 2d French edition of *Leçons d'analyse fonctionelle.* New York, Ungar, 1955. 12+468 pp. $10.00.

For a review of the original edition, cf. this Bulletin, vol. 59, pp. 270–281.

Spheroidal wave functions. By J. A. Stratton, P. M. Morse, L. J. Chu, J. D. C. Little, and F. J. Corbató. New York, The Technology Press of M. I. T. and John Wiley and Sons, 1956. 13+613 pp. $12.50.

Tables of coefficients of the expansions of the radial and angular functions of the first kind in spherical Bessel functions and in associated Legendre functions, computed, tabulated, and printed automatically.