THE NOVEMBER MEETING IN EVANSTON

The five hundred twenty-ninth meeting of the American Mathematical Society was held at Northwestern University, Evanston, Illinois, on Friday, November 23, 1956. Sessions began at 10:30 a.m. and the meeting concluded with a tea at which the ladies of the Department of Mathematics entertained the Society and its guests. There were a total of 155 registrations. Among these there were 138 members of the Society.

The Committee to Select Hour Speakers for Western Sectional Meetings had invited Professor George Piranian of the University of Michigan to address the Society. Professor Piranian spoke on *The boundary of a simple connected domain* at 2:00 p.m. in Lecture Room 2 of the Northwestern Technological Institute Building. Professor Piranian's lecture was unquestionably the high point of the proceedings and was pleasant as well as instructive. Professor L. C. Young presided at the lecture.

There were four sections for the presentation of contributive papers. Presiding officers were Professors Gaffney, Boothby, Gillman, and Auslander.

ALGEBRA AND THEORY OF NUMBERS

88. Philip Dwinger: *Complete reducibility in complete modular lattices with an application to universal algebras.*

An element a of a complete modular lattice L is called characteristic with respect to an automorphism γ of L if $a^\gamma \leqq a$. L is characteristically irreducible with respect to some subgroup $G'[L]$ of the group $G[L]$ of automorphisms of L, if L has no element $\neq 0$ and $\neq 1$, which is characteristic with respect to all the automorphisms belonging to $G'[L]$. An element $a \in L$ is completely reducible if it is a finite sum of minimal elements of L. Theorem I. If L has finite length and L is characteristically irreducible with respect to some subgroup $G'[L]$ of $G[L]$, then every element of L is completely reducible and L is complemented. If $\mathfrak{A}$ is a universal algebra and $G[\mathfrak{A}]$ denotes the group of automorphisms of $\mathfrak{A}$, $C[\mathfrak{A}]$ the lattice of congruence relations of $\mathfrak{A}$ and $G[C[\mathfrak{A}]]$ the group of automorphisms of $C[\mathfrak{A}]$, then $G[\mathfrak{A}]$ is homomorphic to a certain subgroup $G'[C[\mathfrak{A}]]$ of $G[C[\mathfrak{A}]]$. $\mathfrak{A}$ is called characteristically simple if $C[\mathfrak{A}]$ is characteristically irreducible with respect to $G'[C[\mathfrak{A}]]$. If $\mathfrak{A}$ has a selected one-element subalgebra 1, then for every congruence relation $\theta \in C[\mathfrak{A}]$, $S(\theta)$ stands for the set of of all elements $x \in \mathfrak{A}$ for which $x \equiv 1 \pmod{\theta}$. Theorem II. If $\mathfrak{A}$ is an algebra all of whose congruence relations are permutable and with a selected one-element subalgebra 1 and if $\mathfrak{A}$ is characteristically simple and $C[\mathfrak{A}]$ has finite length, then for every congruence relation $\theta \in C[\mathfrak{A}]$, $S(\theta)$ is the finite union of simple isomorphic subalgebras $S(\theta_1), S(\theta_2), \cdots, S(\theta_n)$, every $\theta_i \in C[\mathfrak{A}]$, $i = 1, 2, \cdots, n$. If $\mathfrak{A}$ is a group, then Theorem II yields a well known theorem on direct decompositions in groups. (Zassenhaus, *Theory of groups*, 1949). (Received September 29, 1956.)

89. Casper Goffman: *A lattice homomorphism of a lattice ordered group.*

Let G be a lattice ordered group and L the set of positive elements of G. For every $x\in L$, let $D(x)$ be the set of all $y\in L$ for which $x\cap y=0$. Let α be defined by $\alpha(x)=\alpha(y)$ if and only if $D(x)=D(y)$ and let $\alpha(x)\geqq\alpha(y)$ if $D(x)\subset D(y)$. Jaffard has shown (Journal de Math., 1953) that α satisfies the conditions (a) $\alpha(x)=\alpha(0)$ implies $x=0$, (b) $\alpha(x\cup y)=\alpha(x)\cup\alpha(y)$, $\alpha(x\cap y)=\alpha(x)\cap\alpha(y)$, and (c) $\alpha(x+y)=\alpha(x\cup y)$ so that, in particular, α is a lattice homomorphism. It is shown here that α also satisfies (d) if $x=\sup [x_i|i\in l]$ then $\sup [\alpha(x_i)|i\in l]$ exists and $\alpha(x)=\sup [\alpha(x_i)|i\in l]$. The main result is that if G is archimedean then α is the only lattice homomorphism of L which has properties (a), (b), (c), and (d). An example is given of a nonarchimedean group for which this is false. Examples are given showing the independence of (c) and (d) with each other and with (a) and (b) for lattice homomorphisms. Pierce (Ann. of Math. (1954)) has shown that α is a lattice homomorphism for any distributive lattice with smallest element. A nondistributive lattice is given for which α is not a lattice homomorphism. (Received September 20, 1956.)

90*t*. H. Schwartz and H. T. Muhly: *On a class of cubic diophantine equations.*

The diophantine equation $x^2+y^2+z^2-axyz=b$, has received some attention in the literature where it has unfortunately been the subject of some erroneous statements. In this note these errors are corrected and properties of this equation are investigated. For example, certain solutions from which all others can be derived in a natural way are singled out and termed fundamental. It is shown that except when $a=1$, $b=4$ or when $a=2$, $b=1$, at most a finite number of fundamental solutions can exist, while infinitely many fundamental solutions do exist in these two cases. When $a=1$, $b=2$ there are only a finite number of integer solutions, but otherwise when b is not a perfect square the existence of one solution implies the existence of infinitely many. (Received July 12, 1956.)

91*t*. W. R. Scott: *On the multiplicative group of a division ring.*

Let K be a noncommutative division ring, and let K^* be its multiplicative group. Let Z^* be the center of K^*. Then Hua (*On the multiplicative group of a sfield*, Acad. Sinica Science Record vol. 3 (1950) pp. 1–6) has shown that (i) K^*/Z^* has center 1, and (ii) K^*/Z^* is not solvable. In the present paper, it is shown that K^*/Z^* has no normal Abelian subgroups. This result contains both (i) and (ii). The following generalization is proved. Let G and H be subinvariant subgroups of K^*, $x\in G$, $y\in H$, and suppose that $1\neq[x, y]=z\in Z^*$. Then one of G and H is not nilpotent. (Received October 1, 1956.)

92. W. R. Scott: *Solvable factorizable groups.*

Let H and K be subgroups of a finite group G, and suppose $G=HK$. During recent years, a number of theorems of the following type have been proved: if H and K satisfy certain conditions, then G is solvable. In this paper several additional theorems of this kind are given. It is shown that G is solvable under any of the following conditions: (i) H nilpotent, K Hamiltonian, (ii) H nilpotent of odd order, K contains a subgroup L of index 2 such that all subgroups of L are normal in K, (iii) H cyclic, K contains a subgroup L of index 2 or 3 such that all subgroups of L are normal in K, or (iv) H dihedral or dicyclic, K dihedral or dicyclic. (Received October 1, 1956.)

93*t*. W. R. Scott: *Half-homomorphisms of groups.*

Let G and G' be groups, and let there be given a mapping $a \rightarrow a'$ from G into G'. Call the mapping a *half-homomorphism* if, for all a and b in G, either $(ab)' = a'b'$ or $(ab)' = b'a'$. The following theorem is proved: every half-homomorphism is either a homomorphism or an anti-homomorphism. (Received October 1, 1956.)

94. M. F. Smiley: *Jordan homomorphisms onto prime rings.*

A brief proof is given of the following slight generalization of a theorem of I. N. Herstein (Trans. Amer. Math. Soc. vol. 81 (1956) pp. 331–341). *A Jordan homomorphism* (N. Jacobson and C. E. Rickart, Trans. Amer. Math. Soc. vol. 69 (1950) pp. 479–502) *onto a prime ring* (N. H. McCoy, Amer. J. Math. vol. 71 (1949), pp. 823–833) *is a homomorphism or an anti-homomorphism.* The proof rests on extensions of Herstein's identities and the following simple lemma. *If P is a prime ring in which the square of every commutator is zero, then P is commutative.* (Received October 2, 1956.)

95. M. L. Tomber: *Lie algebras of types A, B, C, D, and F.*

Let Φ be an arbitrary field of characteristic 0 with algebraic closure Ω. Lie algebras $\mathfrak{L}$ over Φ of types A, B, C, D, and F with the exception of D_4 are known to be derivation algebras $\mathfrak{D}(\mathfrak{J})$ of central simple Jordan algebras $\mathfrak{J}$ over Φ. A direct proof of this result is given. It is first shown that any automorphism of $\mathfrak{L}$ over Ω has the form $D \rightarrow SDS^{-1}$ for a unique automorphism S of $\mathfrak{J}$ over Ω. The methods of Jacobson (Duke Math. J. vol. 5 (1939) pp. 775–783) are then used to prove the main theorem and $\mathfrak{D}(\mathfrak{J}_1) \cong \mathfrak{D}(\mathfrak{J}_2)$ if and only if $\mathfrak{J}_1 \cong \mathfrak{J}_2$. (Received October 4, 1956.)

ANALYSIS

96. C. L. Dolph: *A saddle point characterization of the Schwinger variational principles for exterior scattering problems.*

Positive semi-definiteness of the imaginary parts of the free-space Green's function for the exterior Dirichlet and Neumann problems is established. This property is then applied to give global saddle point behavior of the reflection coefficient of the Schwinger variational principles in a manner similar to that appearing in Dolph and Ritt, Math. Zeit. vol. 65 (1956) pp. 309–326. A variant of the usual method of deriving the integral relationships for these problems is also given which illustrates more clearly the relationship between the use of single and double layer for these problems. (Received October 4, 1956.)

97*t*. R. J. Evey and P. C. Hammer: *Compositions of convex and concave functions.*

G. Szekeres recently gave necessary and sufficient conditions that an increasing function with positive derivatives and continuous second derivative be representable as a composition of two increasing functions one of which is concave the other convex. In this paper we establish necessary and sufficient conditions for any increasing function is represented as the composition of a convex function and a concave function. One result is that any continuous nondecreasing function is uniformly approximable by a concave function of a convex function over its entire range, both factors in the composition being piece-wise linear and properly increasing. (Received October 3, 1956.)

98. W. H. Fleming: *Nondegenerate surfaces of finite topological type.*

This paper establishes the following: *Theorem.* Every nondegenerate Fréchet surface in N-space ($N \geqq 2$) of the topological type of a compact 2-manifold, with or without boundary, with finite Lebesgue area has a quasi-conformal representation. For 2-cells and 2-spheres this is Morrey's theorem [Amer. J. Math. vols. 57, 58]. For higher topological types the domain of the quasi-conformal representation is a normalized parallel slit domain, of the sort used by Courant [Dirichlet's Principle, 1950] and Shiffman [Amer. J. Math. vol. 61 (1939)] in connection with the Plateau problem. A surface S is nondegenerate if and only if its middle space is a 2-manifold of the given topological type. (Received September 10, 1956.)

99. R. E. Fullerton: *A geometric characterization of absolute bases in a Banach space.*

Let X be a Banach space. It is shown that if X has a countable basis there exists in X a C-cone K where K is the closure of the convex set determined by a countable set of extreme rays and where K has this further property that there exists $X \in K$ such that $P_x = K \cap (x - K)$ is compact and has its linear extension dense in the space. A necessary and sufficient condition that X possess an absolute countable basis is that there exist $x \in K$ such that P_x has linear extension dense in the space and P_x is linearly homeomorphic to the fundamental cube of Hilbert. (Received October 5, 1956.)

100. M. P. Gaffney: *The asymptotic distribution of the characteristic values and characteristic forms of the Laplacian on a Riemannian manifold.*

Results and methods obtained by Minakshisundarum for functions are generalized to differential forms. It is shown that the fundamental solution of the heat equation (a double p form) exists and equals $\sum_1^\infty \omega_i(P)\omega_i(Q)e^{-\lambda_i t}$. The behaviour of this solution is studied as t approaches zero. The knowledge of this behaviour makes it possible to obtain the asymptotic distributions from Tauberian theorems. (Received October 6, 1956.)

101. W. B. Jurkat: *On the converse of Abel's limit theorem by complex methods.*

The paper contains a direct proof of the well-known statement that (i) $f(z) = \sum a_n z^n = o(1)$ as $z \to 1$, $0 < |1-z|/(1-|z|) = O(1)$ implies $\sum a_n = 0$, if the Tauberian condition (ii) $na_n = O(1)$ is satisfied. The method of proof consists of evaluating the partial sums by a contour integral which can be estimated by proper choice and subdivision of the path of integration. There are several extensions: In condition (i) the variable z may be restricted to real values. The condition (ii) may be replaced by $\sum_0^n |ka_k|^2 = O(n)$. Instead of convergence it is possible to derive Cesàro summability of certain negative orders. (Received November 19, 1956.)

102. G. K. Kalisch: *On similarity and isometric equivalence of certain Volterra operators.*

Unique canonical forms are established for Volterra operators T_K operating on $L_p(0, 1) = \{f\}$ $(1 < p < \infty)$ where $T_K f = \int_x^1 K(x, y)f(y)dy$ with respect to similarity and isometric equivalence. The complex kernels K are subject to various conditions: $K(x, y)=(y-x)^{n-1}L(x, y)$ for positive integral n and $L(x, x)\neq 0$; $L(x, x)$ is real; L satisfies certain regularity conditions. If E is the function identically equal to 1, then T_K is similar to a unique cT_E^n for real c; T_K is isometrically equivalent to a unique T_M where $M(x, y)=(y-x)^{n-1}N(x, y)$, $N(x, x)=c$ (c real) and $\text{Im}\,(N_x(x, x))=\text{Im}\,(N_y(x, x))=0$. The relevance of the above conditions imposed on K is discussed by means of examples. (Received October 4, 1956.)

103. J. H. B. Kemperman: *A property of exponential polynomials.*

Let $f_i(x)$, $a_j(x)$, $b_j(x)$, $(i=0, \cdots, p; j=1, \cdots, q)$, be defined for all real x, such that $\sum_{i=1}^{p} f_i(x+r_iy)=\sum_{j=1}^{q} a_j(x)b_j(y)$ for all x, y. Here, the r_i are $p+1$ different real constants $\neq 0$. Then, provided that $f_0(x)$ is (Lebesgue) measurable, $f_0(x)$ is an exponential polynomial, i.e. a solution of a homogeneous linear differential equation with constant coefficients. This generalizes a result due to St. Kaczmarz, Fund. Math. vol. 6 (1924) pp. 122–129. The above result fails to remain true when "measurable" is replaced by "bounded." (Received October 5, 1956.)

104. Karel deLeeuw: *Linear spaces with compact groups of operators.*

Let A be a complete locally convex topological linear space and G a compact abelian group of operators on A with the map $(x, T_\sigma)\rightarrow T_\sigma(x)$ jointly continuous. For any character χ of G, the map T_χ defined by the vector valued integral $T_\chi(x)=\int_G T_\sigma(x)\chi^{-1}(\sigma)d\sigma$, where $d\sigma$ is Haar measure, is a projection on the subspace $A_\chi=\{x: T_\sigma(x)=\chi(\sigma)x, \text{ all } \sigma \text{ in } G\}$. The *formal topology* on A is the weakest topology that agrees with the original topology on the A and which is such that the T_χ are continuous. In the spaces that occur in analysis, the formal topology is that of termwise convergence of Fourier or power series. *Theorem*: A G-invariant, convex subset of A closed in the original topology is closed in the formal topology. *Corollary*: If A is a Banach space with a G-invariant norm, the unit sphere is closed and the norm is lower semi-continuous in the formal topology. Results of a similar nature hold if G is not abelian. Sample application of theorem: If A is the space of entire functions, and $p(f)$ is the maximum modulus of f in a disk having the origin as center, p is lower semi-continuous in the topology of termwise convergence of power series. (Received October 3, 1956.)

105. Z. A. Melzak: *A scalar transport equation.*

The equation $\partial f(x, t)/\partial t=2^{-1}\int_0^x f(y, t)f(x-y, t)\phi(y, x-y)dy-f(x, t)\int_0^\infty f(y, t)\phi(x, y)dy+\int_x^\infty f(y, t)\psi(y, x)dy-(f(x, t)/x)\int_0^x y\psi(x, y)dy$ with $f(x, 0)$, $\phi(x, y)$, $\psi(x, y)$ known, is shown to possess a unique, continuous, non-negative, $L'(0, \infty)$ in x for each t and analytic in t for each x, solution $f(x, t)$ under the following hypotheses: (1) $f(x, 0)$ is continuous, bounded and non-negative on $[0, \infty)$ and in L' there (2) $\phi(x, y)$ is continuous, symmetric, bounded and non-negative in the first quadrant (3) $\psi(x, y)$ is continuous, bounded, non-negative and satisfies: $\int_0^x y\psi(x, y)dy<x$, $\int_0^x \psi(x, y)dy<c$. The equation itself occurs in a variety of physical problems, hence the non-negativity and boundedness assumptions. The proof proceeds by constructing a power-series solution valid on $0\leqq t\leqq t_1$, $t_1>0$, showing that this solution is non-negative and that $f(x, t_1)$ satisfies the same conditions as $f(x, 0)$, and demonstrating that the continuation process does not terminate. Uniqueness follows from another hypothesis on $\psi(x, y)$. Finally, a condition is given for strict positivity of $f(x, t)$ for $t>0$. (Received September 24, 1956.)

106. C. Y. Pauc: *Theorems of Ward for cell functions.*

A. J. Ward proved the following propositions: (W1) A finitely additive interval function is differentiable in the ordinary sense a.e. on the set at which either of its extreme ordinary derivates is finite. (W2) A finitely additive interval function is differentiable a.e. on the set at which both its extreme strong derivates are finite. D. Rutovitz (in a joint note with C. Y. Pauc in the C. R. Acad. Sci. Paris vol. 240 (1955) pp. 1956–1958) gave a version of (W2) postulating in an abstract measure space $(R, \mathfrak{M}, \mu)$ a directed set (filter) of cell partitions of R, a submetrical norm δ for the cells I, J, a grating axiom and the density axiom. Omitting the density axiom we prove: If ψ is a finitely additive cell function and E a subset of R on which both extreme δ-derivates are finite, there exists a μ-measurable function f ("Denjoy integrant") defined on a measure cover of E with the following property: any set $S\subset E$ of positive outer measure includes a bounded subset L of positive outer measure such that f is essentially bounded on a Jordan cover $\tilde{L}$ of L and $F(I)=\int_{\tilde{L}\cap I} f\cdot d\mu +\int_I \psi(J\|L)$ for any cell I, the last integral being of Burkill type and $\psi(J\|L)=0$ if J intersects L, $=\psi(J)$ if $J\cap L$ is empty. The Denjoy integrant is defined as a fusion of Radon-Nikodym integrants. The theorem is applicable to intervals of any enumerable power of the numerical line or to any enumerable torus space. Moreover f admits a representation as δ-derivative of F is the density axiom holds. D. Rutovitz, in a paper to appear in the Annali di Matematica 1957, will give a version of (W1) for abstract free nets of Jessen type and an extension of other results of Ward's theory of interval functions. (Received September 28, 1956.)

107. I. E. Segal: *Transformation of distributions in Hilbert space.*

Let n be an isotropic normal distribution on a real Hilbert space H, with zero mean (cf. Trans. Amer. Math. Soc. vol. 81 (1946) pp. 106–134). For any closed densely-defined linear transformation T on H, the transform of n by T is absolutely continuous with respect to n if and only if $T*T$ is nonsingular and has the form $I+B$, where B is an operator with absolutely convergent trace. If m denotes the normal distribution on H with covariance operator $(T*T)^{-1}$ and arbitrary mean a in H, m is likewise absolutely continuous with respect to n if T^*T has the preceding form, and the derivative is given by the equation $dm/dn=[\det (I+B)]^{(1/2)} \exp [-\{(Bx, x) -2((I+B)x, a)+((I+B)a, a)\}/2]$. This may be compared with the formula of Cameron and Martin in Trans. Amer. Math. Soc. vol. 58 (1945) pp. 184–219. (Received October 1, 1956.)

108. V. L. Shapiro. *Uniqueness of multiple trigonometric series—An n-dimensional analogue of Verblunsky's theorem.*

In this paper, two theorems on the uniqueness of multiple trigonometric series are proved. The first theorem when considered in one dimension reduces to the well-known Verblunsky result, and the second theorem when considered in one dimension is false. With $n\geqq 2$ and with $x=(x_1, \cdots, x_n)$, $ax+by=(ax_1+by_1, \cdots, ax_n+by_n)$, $(x, y) = (x_1y_1+ \cdots +x_ny_n)$, $|x| = (x, x)^{1/2}$, m an integral lattice point, and $T_n=\{x; -\pi<x_j\leqq\pi, j=1, \cdots, n\}$, the following two theorems are proved: Theorem 1. Given the multiple trigonometric series $\sum_m a_m e^{i(m,x)}$. Set $f_*(x) =\liminf_{t\to 0} \sum_m a_m e^{i(m,x)-|m|t}$ and let $f^*(x)$ designate the corresponding lim sup. Suppose that (i) $\sum_{R-1<|m|\leqq R} |a_m| =o(R)$ as $R\to\infty$. (ii) $f_*(x)$ and $f^*(x)$ are finite for all x. (iii) $\bar{a}_m=a_{-m}$. (iv) $f_*(x)\geqq A(x)$ where $A(x)$ is in L^1 on T_n. Then $f_*(x)$ is in L^1 on T_n and $\sum_m a_m e^{i(m,x)}$ is its Fourier series. Theorem 2. Given the multiple trigonometric series

$\sum_m a_m e^{i(m,x)}$. Let W be a set of measure zero on T_n and q be a point on T_n not in W. Suppose that (i) $\sum_{R-1<|m|\leq R} |a_m| = o(R)$ as $R \to \infty$. (ii) $f^*(x)$ and $f_*(x)$ are finite for x in $T_n - q$. (iii) $f_*(x) = f^*(x)$ for x in $T_n - (W+q)$. (iv) $f_*(x)$ is in L^1 on T_n. Then $\sum_m a_m e^{i(m,x)}$ is the Fourier series of $f_*(x)$. (Received October 3, 1956.)

109t. F. M. Wright: *On Stieltjes mean sigma integrals of order p.*

The author considers a Stieltjes mean sigma integral $(m_p\sigma)\int_a^b f dg$ of order p, where p is an arbitrary integer ≥ 2. If $p=2$, this integral is the Stieltjes mean sigma integral of H. L. Smith (Trans. Amer. Math. Soc., 1925). If g is monotone nondecreasing, and if c is a number such that $a \leq c < b$ and $g(c+) > g(c)$, then $(m_p\sigma)\int_a^b f dg$ does not exist in case f is bounded near c and $f(c+)$ does not exist; a corresponding result holds for left-hand limits. If g is monotone nondecreasing, if $g(x) \equiv g(a)$ for $x < a$ and $g(x) \equiv g(b)$ for $x > b$, and if f is a simple step function, then $(m_p\sigma)\int_a^b f dg$ exists and equals $(LS)\int_a^b f dg + (1-1/p) \cdot \{ \sum [f(x+) - f(x)] \cdot [g(x+) - g(x)] + \sum [f(x-) - f(x)] \cdot [g(x) - g(x-)] \}$. This formula may be used to give a fairly simple proof of a theorem relative to the convergence of a sequence $\{(m_p\sigma)\int_a^b f_n dg\}$, $(n=1, 2, 3, \cdots)$, of such integrals under certain hypotheses. For $p=2$, this theorem includes the principal result of P. Porcelli (Proc. Amer. Math. Soc. (1954) pp. 395–400). (Received October 3, 1956.)

GEOMETRY

110. Lawrence Markus: *Open affine manifolds.*

A complete flat (curvature and torsion zero) affinely connected manifold M^n, which is isomorphic with the affine number space R^n near infinity, is R^n. Similarly a complete Riemannian space M^n of constant negative curvature, which is isometric with its universal covering space B^n near infinity, is B^n. (Received October 4, 1956.)

111. T. S. Motzkin: *Comonotone curves and polyhedra.*

A curve without common points, except the point of contact, with its osculating hyperplanes is called comonotone; if it has no hyperosculating hyperplanes it is strictly comonotone. The convex hull of m points on a strictly comonotone curve in real n-space is a comonotone polyhedron; the m osculating hyperplanes determine a comonotone dissection of projective space (e.g. the dissection of the set of all real polynomials of given degree by a finite set of real numbers excluded as roots). Comonotone dissections and polyhedra and their duals appear to be the most important types definable for every m and n. Among their prominent features are a simple enumeration and characterization of faces of d dimensions $d=1, 2, \cdots$; highest number f of faces for polyhedra, namely $f = \sum_{j=[d/2]+1}^{n/2} (m/j)\binom{m-j-1}{j-1}\binom{j}{d-j+1}$ for even n, $f(n, m, d) = (m+1)/(d+1)f(n+1, m+1, d+1)$ for odd n; greatest skewness of f as function of d, with maximum arbitrarily near to $d=3n/4$; the fact, that every $[n/2]$ vertices are neighbors (form a simplicial face), which distinguishes these polyhedra uniquely for even n; connection with variation-diminishing transformations. (Received October 3, 1956.)

112t. T. S. Motzkin: *Types of dissections.*

The set of all dissections of real projective $(n-1)$-space by $m+n$ hyperplanes is decomposed by the degenerate dissections (those with concurrent hyperplanes) into $c(m, n)$ connected components. It is proved by a 1-1 correspondence between dissections and "transposed" dissections that $c(m, n) = c(n, m)$. We have $c=1$ for min (m, n)

$\leqq 2$; $n+2$ hyperplanes determine uniquely a cyclic order and a norm curve. For $m=n=3$ and $m=3$, $n=4$ the components are well known; for $m=3$, $n=5$ they were listed by the author and A. Benhanan. Fundamental invariants of a dissection, within its component, are its genus g and class k. These are the smallest numbers for which there exist (1) a dissection in the same component with hyperplane coordinate integers absolutely $\leqq k$, including an n by n unit matrix; (2) a curve osculating the given hyperplanes which can be deformed, without ever having hyperosculating hyperplanes into the norm curve $(1, t, \cdots, t^{n-1})$ traced g times. (Received October 3, 1956.)

113*t*. H. T. Muhly: *On the relative arithmetic genus of a normal surface.*

Let F be a normal surface and let P be a point of F. If L is the local ring at P and if m is the ideal of nonunits in L, then L is integrally closed in its field of quotients, but if P is singular, the various powers m^n of m need not be integrally closed in the sense of Prufer (or complete ideals in the sense of Zariski, b-ideals in the sense of Krull). If Q_n is the least complete ideal which contains m^n we show that there is an integer d such that $Q_{nd}=Q_d^n$ for all n. Thus by a theorem of P. Samuel (Thesis, Paris, 1951), the length of Q_{nd} is a polynomial $H(n)$ in n with integer coefficients. If F^* is the normal surface obtained from F by a quadratic transformation with center P followed by normalization, it is shown that the difference $p_a(F^*)-p_a(F)$ between the relative arithmetic genera of F^* and F is equal to the constant term of $H(n)$. (Received October 2, 1956.)

114*t*. A. R. Schweitzer: *Mathematics, related disciplines and Shakespearean interpretations.* Preliminary report.

The author discusses subjects which tend to emphasize Shakespeare's myriad mindedness. These subjects include mathematics, music, philosophy, metaphysics and principles which are assumed to govern an academy ("academe"). Shakespeare mentions the latter subjects in *The taming of the shrew* (Act I, Scene I, Tranio) *The merchant of Venice* (Act V, Scene I, Lorenzo) and *Love's labour lost* (Act I, Scene I, King of Navarre). The term "academy" is assumed by the author to mean an association of persons who unite to promote learning. (Received October 4, 1956.)

115. M. Z. Krzywoblocki: *Examples from statistical mechanics of a continuous medium.*

Kampé de Fériet had provided examples of a statistical mechanics for a continuous medium (Congrès Internat., Philosophie des Sciences, Paris, 1949, vol. 3 (1951) pp. 129–144, and Proc. Sec. Berkeley Symp. Math. Statist. and Probability, University of California Press, 1952, pp. 553–566]: a vibrating string of an infinite length and a vibrating string of finite length with both ends fixed. In the present paper the author furnishes proof of existence of a few more examples of a similar nature. (Received August 24, 1956.)

Topology

116. W. F. Davison: *Mosaics of curves and arcs.*

Let $\{(X_a, T_a): a \in A\}$ be a mosaic of curves (Peano spaces) on X, and let T be the ensuing mosaic topology. (For terminology see W. F. Davison, *Convergent sequences and mosaics*, Bull. Amer. Math. Soc. vol. 62 (1956) p. 180.) Then the *curve space* (X, T) is a mosaic space with the following additional properties: (i) every T-convergent sequence has a subsequence in a curve of the mosaic; (ii) every open subspace is a curve space; (iii) every component is open and arcwise connected; and (iv) the space is locally arcwise connected. A topological space is the curve space of some mosaic of curves if and only if it is mosaic space with property (i) above. If a space is locally separable then it is a curve space if and only if it is Hausdorff and locally arcwise connected. If the mosaic $\{(X_a, T_a): a \in A\}$ consists of arcs then the *arc space* (X, T) has the properties analogous to (i) through (iv) above, obtained by replacing the word "curve" with "arc" throughout, and can be characterized as a mosaic space with the analog of property (i). Consequently, a mosaic space is strongly arcwise connected and has every point covered by an arc if and only if it is a countably compact arc space. (Received October 3, 1956.)

117. O. H. Hamilton: *Fixed points for certain noncontinuous transformations.*

A *connectedness map* is defined by John Nash to be a single valued transformation from a space A into a space B such that the induced map $A \rightarrow A \times B$ preserves the connectedness of connected subsets of A. A *peripherally continuous transformation* is defined by the author to be a single valued transformation of A into B such that if p is a point of A, and V and U are open subsets of A and B containing p and $T(p)$ respectively, then there is an open subset D of V such that T transforms the boundary of D into U. It is shown that a connectedness map of a closed n-cell I into itself is a peripherally continuous transformation. It is shown further that a peripherally continuous transformation of a closed n-cell into itself and hence a connectedness map of a closed n-cell into itself leaves a point invariant. Various examples are presented. (Received September 10, 1956.)

118. J. F. Daly (p) and L. J. Heider: *Generalized G_δ spaces*, II.

Let X denote a completely regular space, while vX denotes the Hewitt Q-space extension of X. Y is called an *imbedded* subspace of vX if Y is dense in vX and every function defined and continuous on Y can be extended as continuous over all of vX. The points of vX included in every *imbedded* subspace of vX are exactly the generalized G_δ points of vX. A generalized G_δ point p of the space X such that every bounded continuous function on $X-\{p\}$ has a continuous extension at p, but such that some unbounded continuous function on $X-\{p\}$ lacks such an extension is, in fact, a G_δ point of X. A nonisolated point p of X is such that every bounded function continuous on $X-\{p\}$ has a continuous extension at p if and only if every finite, open, normal covering of $X-\{p\}$ is the result of deleting the point p from the sets in a finite, open, normal covering of X. (Received September 28, 1956.)

119. L. F. McAuley: *An atomic decomposition of continua into aposyndetic continua.*

Suppose that M is a compact metric continuum and that $p \in M$. Let $M(p)$ denote the set of all points x in M such that there does not exist a collection G of closed separators of M such that (1) there exists an element of G which separates p from x in M and (2) for g in G, a separation $M-g=A+B$, a continuum N in A, and a point b in B, there exist (i) a continuum C containing an open set D which contains N and (ii) three disjoint elements g_1, g_2, and g_3 of G each separating C from b in M and g_2 separating g_1 from g_3 in M. It is shown that $M(p)$ is a continuum for each point p in M. The collection H of all sets $M(p)$ for the various points p in M is an upper semicontinuous collection of disjoint continua filling up M, and furthermore, with respect to its elements as points, H is a compact aposyndetic metric continuum. If M is aposyndetic, then $M(p)=p$ for each point p in M. Results are also obtained for abstract spaces. [Cf. McAuley, Trans. Amer. Math. Soc. vol. 81 (1956) pp. 74–91.] (Received June 18, 1956.)

120. L. E. Pursell: *The ring of all real-valued continuous functions considered as a subring of the ring of all real-valued functions.*

It is shown that if X and Y are completely regular Hausdorff spaces and if there is an isomorphism of R^X onto R^Y which maps $C(X, R)$ onto $C(Y, R)$, then X and Y are homeomorphic. The proof follows readily from the fact that an ideal I in $C(X, R)$ is fixed if and only if there exists g in R^X such that $g(x) \leqq 1$ for all x, $g(x) \neq 1$ for some x, and if f is in I and $f(x) \leqq 1$ for all x, then $f(x) \leqq g(x)$ for all x. (Received October 4, 1956.)

121*t*. Jerome Spanier: *Some effects of the σ-process on almost complex structures.*

Let M^{2n} be a compact, almost complex manifold and let V^{2r} be a compact, regularly-embedded, almost complex submanifold of M (superscripts denote topological dimension). Denote by $\overline{M}$ the *enlargement of* M obtained by replacing V by a bundle W of normal complex projective spaces of complex dimension $n-r-1$, that is, by applying the σ-process of Hopf (Rend. del. Sem. Math., Roma, vol. 10 (1951) pp. 169–182) to the pair (M, V). Then $\overline{M}^{2n}$is an almost complex manifold, and there is a natural mapping f of $\overline{M}$ onto M which is a homeomorphism of $\overline{M}-W$ onto $M-V$, and which reduces to the fiber projection of W onto V. A study is made of the effects of enlargement on the invariants of the pair $(\overline{M}, V)$. Concerning the cohomology groups, it is shown that if $r<n-1$, for each $p \geqq 0$, $H^p(\overline{M})$ is isomorphic to the direct sum of $H^p(M)$ and a certain subgroup of $H^p(W)$. Under the further restriction that $k^*: H^p(M) \rightarrow H^p(V)$ is onto for all $p \geqq 0$, where k: $V \subset M$ is inclusion, $H^*(\overline{M})$ is shown to be isomorphic (as a ring) to a truncated ring of polynomials (coefficients in $f^*[H^*(M)]$) in a two-dimensional class $X \in H^2(\overline{M})$. The Chern classes of the normal bundle of V in M, as well as the characteristic class of the normal (circle) bundle of W in $\overline{M}$ play an important role in these descriptions. Finally, simple formulas relating the tangential Chern classes of M to those of $\overline{M}$ are obtained in case V is a point of M. (Received October 3, 1956.)

122. Hidehiko Yamabe: *Some type of compact transformation groups*. Preliminary report.

Let G be a compact C^2 transformation group on a euclidean space E^d, such that $g(x)^i = \sum_{j,k}^d [b_j^i(g)x^j + (4+|x|^2)\cdot 4^{-1}(C_{jk}^i \xi^j \xi^k + o(|\xi|^2))]$ where $|x|^2 = \sum_i^d (x^i)^2$, $\xi^i = (4+|x|^2)^{-1}(4x^i)$, $|\xi|^2 = \sum_i^d (\xi^i)^2$, $b_j^i(g)$ orthogonal. Applying suitable transformation, we may assume $\sum_j^d \int_G b_j^i(g^{-1}) c_{hl}^j(g) dg = 0$. v represent translations defined by vector (v). If g and h are two elements in G, $\lim_m (gv^{1/m}g^{-1}hv^{1/m}h^{-1})^m$ define a homeomorphism denoted by $gvg^{-1} \oplus hvh^{-1}$. Take the integral $J[v] = \oplus \int_G gvg^{-1}dg$ in this sense. $J[v]$ can be well defined, $v \rightarrow J[v]$ is univalent and $\{J[v]p$, for all $v\} = E^d$. From this we can construct a homeomorphism Φ from E^d onto E^d, such that $\Phi G \Phi^{-1}$ is orthogonal. From this follows that C^2 compact transformation groups on euclidean spheres with a fixed point is equivalent to an orthogonal transformation group. This is the completion for the statement the author made in winter meeting 1954 with incomplete proof. (Received October 15, 1956.)

J. W. T. Youngs,
Associate Secretary