necessary to show that the group of nonsingular matrices will do it. This is not exactly trivial and requires some such device as Weyl's principle of irrelevance of algebraic inequalities. A rather unusual and commendable feature of the book is the treatment of the rotation group by the method of Stiefel. The spin representations are obtained in several ways, including the method of Brauer and Weyl. The author's handling of the topological matters relevant to the rotation group is perforce rather sketchy, as he readily admits. However it could be made somewhat less so with very little extra effort. For instance, it would be no trouble to give at least a precise analytic definition of homotopy before constructing covering groups. The final chapter is devoted to the finite-dimensional representations of the Lorentz group.

The subject of which this book treats is a difficult one and it is far from easy to achieve clarity of exposition in dealing with it. In the reviewer's opinion, Professor Boerner has succeeded in this respect better than anybody so far. Not the least of the consequences is that it renders more accessible the rest of the literature on the subject, especially Weyl's book which even after all the years since its publication still contains such a wealth of stimulating ideas. There is only one major regret, namely that the symplectic group has been ignored, presumably because of the absence of applications to physics. It is sincerely to be hoped that it will come in for its proper share of attention in a future edition.

W. E. JENNER

The convolution transform by I. I. Hirschman and D. V. Widder. Princeton, Princeton University Press, 1955. 10+268 pp. $5.50.

At a time when so much mathematics writing has had a tendency to become ponderous and unreadable it is a pleasure to have this excellent book appear. The style of the authors has become well known through their extensive papers and the precision and care that has marked their writing in the past is scrupulously maintained throughout the book. Furthermore the subject is itself of considerable interest being classical in every sense of the word, and appearing again and again in the most varied contemporary contexts. The problems treated are solved on their own ground and the theorems are not based on changing the problem. The ratio of theorems to definitions in this book is high.

The main part of the book is devoted to a study of the convolution equation whose kernel has the property that the reciprocal of its Laplace transform is entire and of the form $\exp(-cs^2+bs)\prod(1-s/a_k)\cdot\exp s/a_k$ where a_k is real, $c\geqq 0$ and $\sum a_k^{-2}<\infty$. This class of kernels has been studied extensively in recent years by Schoenberg and some of his work is reproduced in Chapter IV. Except for this the first seven chapters are devoted to the authors' own investigations of the case $c=0$. Chapters VI and VII have detailed discussions of the inversion and representation theories. Chapter VIII contains a discussion of the Weierstrass transform. This is the only case in which $c\neq 0$ that is discussed in the book, although it is actually the whole class described above that appears in several connections as the "natural" one. Chapter IX is concerned with the complex inversion of convolutions whose kernel has a Laplace transform whose reciprocal is of the form $\prod(1-s^2/a_k^2)$ with real a_k and $\lim k/a_k^2=\Omega$, $0<\Omega<\infty$. The final chapter has a discussion of Bernstein polynomials, the asymptotic behavior of the convolution transform, the analytic behavior of the transforms of the kernels and a theorem on quasi-analyticity.

In some respects the title of the book is misleading. While agreeing that it would be a mistake to try to include all of the work in convolution transforms that has been done one cannot help but feel that what is here presented is actually so restricted in scope as to present a distorted view to the reader studying the material for the first time. Moreover the existence of other problems in the field is not even hinted at. Even if it had to be at the expense of some of the finer details in, say the representation or inversion theory, a chapter devoted to a discussion of the state of other questions such as spectral synthesis, equations of the Wiener-Hopf type or local and non-local inversions would have been valuable, particularly from these authors.

In this connection the bibliography is also somewhat inadequate. Even papers of basic importance to the matters discussed are not mentioned. For example, although there is a chapter devoted to the Weierstrass transform the only references to work other than their own is one to Hille's book on semi-groups and one to Tychonoff's paper on his theorem of uniqueness for solutions of the heat equation. Also Pollard's basic paper on the material of Chapter IX (Ann. of Math. vol. 49 (1948) pp. 956–965) is not mentioned although other papers of his of less relevancy are. As a final example Post's work on differential operators (Trans. Amer. Math. Soc. vol. 32 (1930) pp. 723–781) is not listed in spite of the fact that it is not only important for the work already done in the field of convolutions but also shows promise of being important in some of the unsolved problems.

The above comments should not be construed as meaning that the book is not an excellent one. It is just not possible to satisfy everyone on all points.

J. BLACKMAN

Handbuch der Laplace-Transformation, Vol. III. *Anwendungen der Laplace-Transformation*, Part 2. By Gustav Doetsch. Basel and Stuttgart, Birkhäuser, 1956. 300 pp. 40 Swiss francs.

This final volume of Doetsch's treatise contains applications to partial differential equations, difference equations, integral equations, and entire functions of exponential type. (Vols. 1 and 2 were reviewed in this Bulletin, vol. 58 (1952), pp. 670–673, and vol. 62 (1956), p. 628.) An appendix gives a number of remarks supplementing various points in vol. 1. An extensive bibliography for vols. 2 and 3 appears at the end of vol. 3.

For differential equations, the author continues to give a presentation that is mathematically satisfying and at the same time maintains contact with the physical background. At the end of the section on differential equations there is a chapter on "Huygens's and Euler's principles": the former is regarded as a source of relations among special functions which appear as Green's functions; the latter is the author's term for the fact that one can sometimes solve the same equation in two ways and thus obtain an identity connecting the solutions. There are three chapters on difference equations. The first deals with the equation $c_1Y(t)+ \cdots +c_nY(t+n)=G(t)$, where the "initial conditions" are that $Y(t)$ is given in $0\leqq t<n$; this is an unfamiliar analogue of the linear differential equations that are usually treated by transforming the whole equation, and is given an analogous treatment here. The second chapter deals with difference equations in analytic functions, which are treated essentially by applying the inverse Laplace transform. The third chapter gives some examples of how partial difference equations can be solved. The chapters on integral equations take up various types which can be attacked by various methods, including that of Wiener and Hopf; the equation of renewal theory, Abel's equation, and derivatives and integrals of fractional order illustrate the general methods. Special chapters are devoted to functional relations for special functions, obtained from algebraic relations for their transforms. Convolution integral equations (where the convolution is an integral either over a vertical line or a closed path) are studied in the complex domain, again with applications to derivatives and integrals of fractional order. It is to be hoped, but hardly expected, that would-be writers on the subject of differentiability of fractional order will consult the