book is not an excellent one. It is just not possible to satisfy everyone on all points.

J. BLACKMAN

Handbuch der Laplace-Transformation, Vol. III. *Anwendungen der Laplace-Transformation*, Part 2. By Gustav Doetsch. Basel and Stuttgart, Birkhäuser, 1956. 300 pp. 40 Swiss francs.

This final volume of Doetsch's treatise contains applications to partial differential equations, difference equations, integral equations, and entire functions of exponential type. (Vols. 1 and 2 were reviewed in this Bulletin, vol. 58 (1952), pp. 670–673, and vol. 62 (1956), p. 628.) An appendix gives a number of remarks supplementing various points in vol. 1. An extensive bibliography for vols. 2 and 3 appears at the end of vol. 3.

For differential equations, the author continues to give a presentation that is mathematically satisfying and at the same time maintains contact with the physical background. At the end of the section on differential equations there is a chapter on "Huygens's and Euler's principles": the former is regarded as a source of relations among special functions which appear as Green's functions; the latter is the author's term for the fact that one can sometimes solve the same equation in two ways and thus obtain an identity connecting the solutions. There are three chapters on difference equations. The first deals with the equation $c_1Y(t)+\cdots+c_nY(t+n)=G(t)$, where the "initial conditions" are that $Y(t)$ is given in $0\leqq t<n$; this is an unfamiliar analogue of the linear differential equations that are usually treated by transforming the whole equation, and is given an analogous treatment here. The second chapter deals with difference equations in analytic functions, which are treated essentially by applying the inverse Laplace transform. The third chapter gives some examples of how partial difference equations can be solved. The chapters on integral equations take up various types which can be attacked by various methods, including that of Wiener and Hopf; the equation of renewal theory, Abel's equation, and derivatives and integrals of fractional order illustrate the general methods. Special chapters are devoted to functional relations for special functions, obtained from algebraic relations for their transforms. Convolution integral equations (where the convolution is an integral either over a vertical line or a closed path) are studied in the complex domain, again with applications to derivatives and integrals of fractional order. It is to be hoped, but hardly expected, that would-be writers on the subject of differentiability of fractional order will consult the

author's discussions and heed his warning that there cannot be any universal definition.

The last two chapters deal with finite Laplace transforms and entire functions of exponential type. Particularly noteworthy is the most elementary proof yet given of the theorem about the indicator diagram of a finite Laplace transform (the author seems to be unaware that the equivalent result for Fourier transforms was already known).

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