Integral equations and their applications to certain problems in mechanics, mathematical physics and technology. By S. G. Mikhlin. Translated from the Russian by A. H. Armstrong. New York, Pergamon Press, 1957. 12+338 pp. $12.50.

This is a substantial but not an exciting account of the theory of integral equations of the regular and singular type with applications to the physical sciences. The book is divided into two parts, the first third giving the theory and the second two-thirds discussing the applications.

The regular cases, which include those equations whose kernels may be made regular by a finite number of iterations, are studied by the classical methods of Fredholm, Hilbert and Schmidt. Various devices for determining characteristic values are considered and some numerical examples are given. In the last quarter of this section a brief account is given of the type of singular integral equation which the author needs in some of his applications. That is, this final chapter on integral equations discusses those equations whose kernels are of the Cauchy or Hilbert type with the related function theoretic methods required to obtain solutions for these equations. This account is readable and the reader will not be annoyed with the masses of special detail which other writers have attempted to supply on this topic. The stage has now been set to treat boundary problems of the Hilbert-Riemann type which arise so often in the applied fields.

An interesting assortment of problems involving integral equations which arise in the mechanics of continuous media and which come from such diverse equations as Laplace's, the biharmonic, the wave and the diffusion equation (all in two dimensions) is discussed with the machinery developed in the first part of the book. Like his compatriot, Muskhlishvili, Mikhlin refers to unusual problems in wave motion which integral equations can handle and then fails to provide the reader with some of the important details.

ALBERT E. HEINS

Elements of the theory of functions and functional analysis, Vol. 1, *Metric and Normed Spaces*. By A. N. Kolmogorov and S. V. Fomin. A translation by L. Boron of *Elementy Teorii Funkciĭ i Funkcional'nogo Analiza.*, I. *Metričeskie i Normirovannye Prostranstva*. Rochester, Graylock Press, 1957. 9+129 pp. $3.95.

This little book, the first in a projected series by the same authors, is a textbook prepared from material presented at the Moscow State University and could serve as a text for, or as a welcome adjunct to, a semester (or quarter) course in functional analysis for beginning graduate students. The following list of chapter headings and principal topics will indicate its scope.

Chapter 1, devoted to set theory, contains no surprises. We begin with a brief discussion of the concept of set and progress through the usual sequence of topics, viz., set operations, maps, cardinal numbers, equivalence relations, equivalence class decompositions and their associated projections. There are several illuminating examples and exercises. Order relations are not mentioned. Chapter 2 develops the theory of metric spaces and is noteworthy in several respects, both for what is included and for what is not. We have, of course, sequences and limit points, closed and open sets, continuity and isometry, separability, completeness and completion. There is a special section devoted to the structure of open sets on the real line. The usual discussion of uniformity is simply omitted; the concept of a topological space is introduced but plays no further role (nonmetrizability is illustrated in a 2-point space); the category theorems are deferred; G_δ's do not put in an appearance. In exchange for these more familiar discussions, we are offered several interesting and relatively unfamiliar ones. As early as possible, the authors prove the theorem that a properly contracting map on a complete metric space has a unique fix-point, and proceed, at once, to exploit the principle to the hilt in a number of excellently chosen examples, each worked out in detail. The examples include, most notably, successive approximation to the solution of a system of linear equations with sufficiently dominant main diagonal, integral equations with continuous kernel, and the Picard-Lipschitz existence theorem. Chapter 2 also contains an unusually thorough treatment of compact sets in metric spaces; included is the criterion for the compactness of a set in a $C(X, Y)$ (Arzela's theorem), also illustrated in the context of differential equations—this time Peano's existence theorem is proved in detail. The chapter closes with a long and interesting, if not always quite lucid, discussion of arc-length in a metric space, including a proof of the existence, under suitable conditions, of a shortest arc between two points. Chapters 1 and 2 together occupy slightly over half of the book and seem to the reviewer to comprise by far the most stimulating and successful portion thereof.

Chapter 3 takes up the theory of normed spaces. After a brief section on convex sets, the usual topics surrounding the notion of linear functional are developed. The Hahn-Banach theorem is stated generally but proved in the separable case only. The dual space, the second dual, the notions of reflexivity and weak convergence of sequences of elements and functionals are introduced and briefly illustrated and discussed. Direct sums, quotient spaces, bases are not mentioned. Neither are the weak topologies as such. (Neither, mysteriously, is the completion of a normed space, though the completion of an arbitrary metric space was constructed in Chapter 2.) The chapter closes with a section on linear transformations which develops the basic facts concerning boundedness, algebraic combinations, adjoints and inverses of operators. After Chapter 3 is interpolated a five page appendix giving an illuminating account of the elements of the theory of generalized functions (distributions). The fourth and final chapter on linear operator equations is quite brief. Attention is sharply focused on the compact case. The Fredholm theorems are treated, and it is shown that a usefully broad class of mildly discontinuous kernels defines compact operators. In contrast with the wealth of illustrative material in Chapter 2, the examples in Chapters 3 and 4 become gradually scarcer and more routine until, at last, they peter out altogether; the Fredholm theorems are unaccompanied by either example or exercise.

Attention has already been called to the absence of some topics that would be thought by many to form a natural or even an indispensable part of a book on the "elements" of functional analysis. The list could be lengthened. Lebesgue integration is not once mentioned (though the foreword promises its appearance in a later volume). Neither is Stieltjes integration. (The only linear functionals on $C(X)$ that appear are atoms.) Even such a work horse as the Banach-Steinhaus theorem is not to be found. These omissions of more or less standard material are not cited as errors of judgment or faults of the book, but rather to indicate its spirit. Indeed, its title is one that would mean different things to different mathematicians. The authors of the present book have attempted to assemble, in compact, consequential, and conceptually unified form, those things that will prove most useful to the reader in subsequent encounters with the methods of functional analysis in applied mathematics; and they judge accordingly that, say, the Fredholm alternative and the Neumann series are elemental, while a constructive characterization of the L_2 completion of the space of continuous functions is an interesting sidelight that may safely be postponed. This point of view is, regrettably, foreign to much current instruction in functional analysis, at least in this country, and the present book is quite unlike any text heretofore available in English. (On the other hand, it is quite close, both in spirit and content, to the earlier and more extensive treatise with the same title by Sobolev and Lusternik.) Its translation is opportune and will be welcome to students of mathematical physics as well as to students of mathematics.

There are several errors in the book—some troublesome, some not. In the latter category, it is perhaps worth mentioning that the assertion about the reflexivity of $\bar{E}$ (p. 90, line 13) should, of course, be restricted to complete spaces. (Even so restricted the statement is unclear at best; R. C. James has shown that E and $\bar{\bar{E}}$ may not be "distinct" even if E is not reflexive.) The proofs offered for two of the main Fredholm theorems (Theorems 2 and 3, pp. 119–120) are inadequate, but the errors betray themselves at once and the reader will have no trouble finding correct proofs elsewhere. On the other hand, the proposed metrization of the weak*-topology on the entirety of an infinite dimensional dual space (p. 95) is likely to confuse the student, as is the mistaken assertion (p. 92) that linear combinations of atoms are uniformly dense in the dual of $C[0, 1]$. (They are weakly dense, of course.) The special role of 0 in the spectrum of a compact operator is overlooked (p. 116 and again p. 120). The proof (p. 65) that a function uniformly close to a function of unbounded variation has itself unbounded variation is (necessarily) fallacious. Finally, the reviewer disputes the assertion that the equivalence relation introduced on p. 68 is obviously transitive.

With few exceptions the translation is *verbatim*, and is for the most part felicitous. A very few liberties have been taken with the text, to correct an oversight or to bring a definition into agreement with custom; the usual Bunyakovski→Schwarz transformation has been carried out; topicalities have been removed, so that, for instance, the "set of all automobiles in Moscow" has become the "set of all automobiles in a given city." On the whole one inclines to object that the translation is too literal. The errors mentioned above have been carefully preserved, and, on at least one occasion, even a misprint is reproduced. The translator has enlarged the list of references to include a number of books in English and has provided lists of symbols, of theorems and of definitions (the last sadly incomplete).

In the original version the authors employed, in a familiar fashion, two sizes of type; one, regular, for the main thread of the book, and a smaller one reserved for remarks, examples etc. ancillary to the main development and not necessary to its understanding. Shorter asides were relegated to footnotes. In the present edition the distinction between type faces has been completely ignored and all the footnotes have been thrust abruptly into the text. This, of course, robs the reader of the authors' opinion of the relative importance of the various parts of the book and results in making it considerably harder to read. What were, before, carefully articulated discussions have become, somehow, amorphous. It is hard to imagine that the saving in cost is worth the loss incurred.

ARLEN BROWN

The topology of fibre bundles. By Norman Steenrod. Princeton, Princeton University Press, 1951, second printing, 1957. 8+227 pp. $5.00.

This second printing of Steenrod's well known book differs from the first only in the addition of an appendix which describes progress in the field between 1951 and 1956. The rate of progress has been very high indeed, so that even this appendix requires amendment to bring it up to date.

Today as in 1951 the term "fibre space" is ambiguous: the definitions due to Serre, Hu, and Hurewicz all have a good claim to the title. However, thanks to Steenrod's book, the "fibre bundle" is now a familiar and well defined object. In the applications of topology to differential geometry, and lately also to algebraic geometry, the fibre bundle is a tool of fundamental importance.

The following is a brief description of the book. Part I sets a foundation for the study of fibre bundles. Concepts such as cross-section, bundle map, induced bundle, and principal bundle are defined and studied with the author's characteristic thoroughness. A number of important examples are considered: tensor bundles, covering spaces, the principal bundle over a coset space, and so on. It is shown that any cross-section of a differentiable bundle can be approximated by a suitably differentiable cross-section. As is pointed out in the appendix, many of the theorems of Part I can be technically improved. However this remains the best presentation of the subject matter which is available.

Part II studies the homotopy theory of bundles. The homotopy sequence of a bundle is defined; a classification theorem for bundles over the n-sphere is proved; the theory of universal bundles is developed; and the Hopf fiberings are studied. A number of results about the homotopy groups of spheres and other standard manifolds are obtained. However this last material has been outclassed by subsequent developments in the field. [For recent work see: R. Bott, *The stable homotopy of the classical groups*, Proc. Nat. Acad. Sci. U.S.A. vol. 43 (1957) pp. 933–935; F. Adams, *On the structure and applications of the Steenrod algebra*, Comm. Math. Helv., to appear; and G. F. Paechter, *The groups* $\pi_r(V_{n,m})$ (I), Quart. J. Math. Ser. (2) vol. 7 (1956) pp. 249–268.] Finally the tangent bundle of the n-sphere is studied. [For recent work see forthcoming papers by Kervaire