about the sets introduced by Carleson and Helson which, on account of the questions remaining still open, raise interesting problems for the research student.

RAPHAEL SALEM

Tauberian theorems. By Harry Raymond Pitt. Tata Institute of Fundamental Research Monographs on Mathematics and Physics, vol. 2, Oxford University Press, 1958. 10+174 pp. 30 sh.

The terminology used in this book is well known. A theorem which asserts that, for the transformation

$$g(u) = \int_{-\infty}^{+\infty} k(u, v)s(v)dv, \tag{1}$$

$s(v) \to a$ as $v \to +\infty$ implies $g(u) \to a$ as $u \to +\infty$ is an *Abelian* or a direct theorem. A *Tauberian* or an inverse theorem asserts that conversely if $g(u) \to a$ and if $s(v)$ satisfies some additional condition, the so called Tauberian condition, often of the type that $s(v)$ changes slowly with v, then $s(v) \to a$. A *Mercerian* theorem is an inverse theorem which holds without a Tauberian condition. A theorem is called *special* if it refers to a specific kernel k, and *general* if it holds for an extensive class of kernels.

Tauberian theorems obtained by N. Wiener some 25 years ago, and subsequent contributions of the author, play a central rôle in the whole theory. They are the main subject of this book. Accordingly, readers will find, for example, little about estimation of Tauberian constants, about Tauberian theorems of function theoretic type, asymptotic theorems, best Tauberian conditions and about application of Banach algebras or of locally convex spaces.

The content is as follows: Chapter I–III contain a discussion of very general Tauberian conditions, of slowly decreasing functions; this is followed by elementary general Tauberian theorems and theorems in which boundedness of $g(u)$ implies that of $s(v)$. Special Tauberian theorems are given for the methods of Cesàro, Riesz, Abel and Borel; in the last two cases the proofs furnish also the corresponding high-indices theorems.

Chapters IV and V constitute the main part of the book. After theorems from harmonic analysis about the properties of an analytic function of the Fourier transform $K(t)$ of $k(t)$, the main theorem is proved: if the kernel of (1) is $k(u-v)$, if $K(t) \neq 0$ and if $s(v)$ is bounded, then $S \leqq \epsilon + C(\epsilon)G$, where $S = \lim\sup |s(v)|$, $G = \lim\sup |g(u)|$. Other classical Wiener theorems follow easily. Refinements of these are then discussed: Tauberian theorems where $K(t) \neq 0$ is assumed only for a part of $(-\infty, +\infty)$, theorems where $s(x)$ is assumed bounded from one side at the expense of the positiveness of one of the kernels involved, theorems with abstract metrics instead of $M'(f) = \sup_x |f(x)|$, $M(f) = \lim \sup_{x\to\infty} |f(x)|$. Important for applications are kernels of forms other than $k(x-y)$ but approximable by such kernels. In the main theorem of this type the approximation is in the sense $\sup_{\sigma_1\leq\sigma\leq\sigma_2} \int |k(x, x-y) - k(y)| e^{-\sigma y} dy \to 0$ for $x\to\infty$, while $s(x)$ is allowed to satisfy $s(x) = O(e^{\sigma x})$. Chapter IV ends with the applications to special methods, especially Hausdorff's. A remark on p. 92 is not quite correct: the exact form of the high-indices theorem for the Euler method is known (Meyer-König and Zeller, Math. Z. vol. 66 (1956)). Mercerian theorems in the next chapter are based on the following result about Fourier transforms. Let $k_1 \in V$ if $k_1(t) = \int_{-\infty}^{+\infty} e^{-itx} dk(x)$, with $k(x)$ of bounded variation. Then $k_1(t)^{-1} \in V$ if $\int |dq| \leq \inf_t |D(t)|$, where q is the singular component of k, and D is the Fourier transform of its discontinuous component. Resulting Mercerian theorems are applied to special methods, in particular again to Hausdorff methods with the condition $a_n = O(n^{-1/2})$.

Chapter VI stands by itself. All known proofs of the prime number theorem contain Tauberian ideas, and the author makes it his aim to present these comprehensively. After an exposition of the Landau-Ikehara theorem, "non-elementary" proofs are given, based on it or on a Tauberian theorem for the Lambert method. Wright's modification of Selberg's formula and general Tauberian theorems for the transformation $s(x) + x^{-1}\int_1^\infty s(x-y)dk(y)$ lead to "elementary" proofs.

The author writes with great elegance, lucidity and an unerring taste; many original proofs are given. For a long time to come, this excellent book will remain a standard work of the subject.

G. G. LORENTZ

Die Klassenkörper der komplexen Multiplikation. By M. Deuring. Enzyklopädie der Mathematischen Wissenschaften, Band I_2, Heft 10, Teil II. Stuttgart, Teubner, 1958. 60 pp. DM 15.

The work on complex multiplication associated with the elliptic and modular functions belongs to one of the most beautiful, fascinating and important parts of pure mathematics. It reveals the closest and most surprising connection between complex function theory, algebra and the higher arithmetic, in particular classfield theory; and has also led to the recent developments in the subject.

The topics treated in the booklet have become classical and the author makes use of the various accounts to be found in the treatises