

on algebra by Weber, Fricke and in the book of Fueter on complex multiplication as well as papers by Hasse and himself. The tract contains a fairly rapid and condensed account of the subject and so it requires considerable knowledge by the reader if he is to comprehend fully the subject matter. Nevertheless he will find it very convenient to have this excellent and authoritative production at his disposal.

The book consists of four parts. The first contains the pertinent results on the modular and elliptic functions; and the second the relevant theory of complex quadratic fields and ideal and divisor theory. There are various methods of defining classes of ideals, and the vital problem is to find the classfields, i.e. the Abelian algebraic extensions of the complex quadratic field and their relation to the ideal class groups. This is done in the third and fourth parts in which it is shown that certain modular functions define the classfields for appropriate ideal classes. Two different proofs are given, the first depending on the general theory of Abelian fields while the second is independent of the general classfield theory.

L. J. MORDELL

*Topologie algébrique et théorie des faisceaux.* By Roger Godemont. Actualités Scientifiques et Industrielles, no. 1252, Paris, Hermann, 1958. 8+283 pp.

"We have therefore tried to write a book which presupposes no familiarity with algebraic topology . . ." This statement is taken from the preface of the book under review, and surprisingly enough, the author may have succeeded in his attempt. Of course, until the appearance of the second volume of this work, the reader may find himself still unfamiliar with algebraic topology (at least as the algebraic topologists know it), but the author seems to be very much aware of this deficiency and will probably remedy it soon. The essential fact, however, is that this book starts from scratch (i.e., from approximately the second year graduate school level) and presents extremely lucidly a comprehensive theory of sheaves.

The book is divided into two chapters. The first is devoted to homological algebra, the second to the theory of sheaves. In the first chapter, the standard material on abelian categories, functors, exact sequences, chain complexes, and homology is covered. In addition to this, the author includes simplicial chain complexes and semi-simplicial chain complexes (the semi-simplicial complexes of Eilenberg-Zilber), local coefficients, a concise but clear exposition of spectral sequences, and a study of the functors Ext and Tor. Cartesian and

cup products are taken up here, as well as the theorem on acyclic models. Throughout this first chapter the author exhibits a remarkable talent both for restraint and lucidity. He has managed to select just those parts of homological algebra which are necessary for the development of algebraic topology and sheaf theory, and to present them concisely and not too pedantically.

It is in the second chapter, though, that this book makes its largest contribution. Although notes on sheaf theory have been mimeographed and numerous papers using sheaf theory have been published, this is the first time that a full treatment of the theory of sheaves—starting with the definition and working through to a general homology theory—has appeared in such form that a non-specialist (in particular a fresh graduate student) could learn it. Moreover, the author has taken a different point of view from that taken by Cartan, say, with his fine sheaves or by Grothendieck, with his injective resolutions. To the author, the central problem in sheaf theory is that of extending sections. He therefore defines a sheaf  $F$  over a space  $X$  to be *flabby* (flasque) if every section of  $F$  over any open set of  $X$  can be extended to all of  $X$ . Every sheaf can be imbedded in a flabby sheaf, in fact in a canonical way. For paracompact spaces, the weaker notion of *soft* sheaves (faisceaux mous) is introduced. A sheaf is soft if every section over a closed subset of the space  $X$  can be extended to  $X$ . The soft sheaves replace the fine sheaves of the older theory.

Using a canonical flabby resolution of a sheaf  $F: 0 \rightarrow F \rightarrow C^0 \rightarrow C^1 \rightarrow \dots$  (each  $C^i$  a flabby sheaf, and the sequence is exact), the author defines  $H^n(X; F)$  to be the cohomology groups of the complex

$$0 \rightarrow \Gamma(C^0) \rightarrow \Gamma(C^1) \rightarrow \dots$$

where  $\Gamma(C^i)$  is the group of sections of  $C^i$  over  $X$ . These cohomology groups are defined for any space  $X$ , and are shown to have all the usual properties of a cohomology theory. Using spectral sequences, one obtains in particular canonical homomorphisms  $H^n(\Gamma(\mathcal{L}^*)) \rightarrow H^n(X; F)$  where  $\mathcal{L}^*$  is any resolution of the sheaf  $F$ . If  $\mathcal{L}^*$  is a flabby resolution, these homomorphisms are isomorphisms.

The Čech cohomology groups are introduced, and a spectral sequence relating the Čech cohomology groups to the ones defined above is obtained. Sufficient conditions are given for establishing an isomorphism between the Čech groups and the others (e.g. if the space is paracompact, or an algebraic variety). The second chapter also includes a study of cartesian and cup products in the cohomology theory of sheaves, and ends with a discussion of derived functors.

Since injective sheaves are necessarily flabby, it is seen that the cohomology theory defined in this book agrees with that defined by Grothendieck.

As was said before, this book was clearly and carefully written. As is usually the case with a reference book, however, it reads like one. For one who knows in which direction all this material is heading, the clarity of presentation makes the reading not unpleasant. For students, I think that this book would best be used in conjunction with a seminar, a course, or dextrodine. I fervently hope that volume II of this work will appear soon. If it is as well written as this volume, I think that the author will have done us all a great service.

The table of contents follows:

#### Chapitre I. Algèbre Homologique

1. Modules et foncteurs
2. Généralités sur les complexes
3. Complexes simpliciaux
4. Suites spectrales
5. Les groupes  $\text{Ext}^n(L, M)$  et  $\text{Tor}_n(L, M)$

#### Chapitre II. Théorie des Faisceaux

1. Faisceaux d'ensembles
2. Faisceaux de modules
3. Problèmes de prolongement et de relevement de sections
4. Cohomologie à valeurs dans un faisceau
5. Cohomologie de Čech
6. Produit cartésien et cup-produit
7. Foncteurs dérivés en théorie des faisceaux

Appendice—Résolutions simpliciales standard.

DAVID A. BUCHSBAUM

*Éléments de mathématiques.* By N. Bourbaki. Book II, *Algèbre*. Chapter VIII, *Modules et anneaux semi-simples*. Actualités Scientifiques et Industrielles, no. 1261, Paris, Hermann, 1958.

In the volume under review the polycephalic author has written a beautifully clear exposition of that part of the structure theory of associative rings that he feels is of the greatest interest in itself and also of widest applicability in other branches of mathematics. Although the main stress is on rings with minimum condition, full advantage has been taken, as will be seen below, of the recent work on the structure theory of general rings to simplify and clarify the more classical results.