

the theory much simpler in case $n=2$. A countability result, false for $n>2$, delayed the development of the general theory for many years. The authors go on to consider the concepts of bounded variation and absolute continuity introduced by Tonelli (the first attempt to generalize the classical notions of real variables) and investigate the interconnections with their own ideas. For the benefit of those interested in the field, an unproved conjecture is stated on page 433. From the point of view of applications, it is interesting to note that any Dirichlet transformation is eAC .

So much for a part by part summary. In general the book leaves something to be desired in a nonmathematical way.

The index is inadequate, and it is difficult to follow up the cross references, which are given in the form III.3.3 while the clues at the top of the page cut matters short with III.3. The printing does not live up to the high standards one has been led to expect of Springer-Verlag, and there are numerous broken characters and smudges (see, for example p. 212). The theorems are often easy to miss because their statements are not italicized. Finally the cohomological diagrams should have been left to the mercy of a printer (even a careless printer). Though they were apparently drafted with care, the results are sometimes shocking. All in all, though, these are probably petty grievances—the mathematical content certainly stands on its own feet, and is an outstanding source of information on an important and very difficult portion of modern mathematics.

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Leçons d'algèbre moderne. By P. Dubreil and M. L. Dubreil-Jacotin. Dunod, Paris, 1961. 11+393 pp. 35 NF.

The authors present a course in abstract algebra (the term "modern" is open to question, since almost all the material was already in the first edition of van der Waerden) designed for a program roughly at the level of the master's degree in this country. Their treatment is straightforward, usually aiming for proofs which involve the least complicated apparatus. This approach makes it possible to include a large number of results on assorted topics in fewer than four hundred pages.

Several notions which play an important part in modern mathematics, among them those of module and tensor product, are either given only passing notice or left out completely. These omissions may not be of great importance in the case of students completing their training with this course, but they are serious deficiencies for students

planning further mathematical study. The total absence of exercises is a more deplorable drawback for all who are trying to learn the subject. For the nonspecialist who wishes to consult proofs of basic theorems in algebra, this book should be a reliable reference because of its numerous results and their ready accessibility.

The book begins with a chapter on general binary laws of composition, including the notion of Moore families of subsets (e.g., the family of subgroups of a group) and the behavior of these binary compositions relative to equivalence relations. Later chapters on semigroups and groups treat the fundamental homomorphism and isomorphism theorems for groups (with operators), Sylow theory, free groups, generators and relations. The Jordan-Hoelder theorem and the structure of finitely generated abelian groups are established; the Krull-Schmidt theorem (except in the case of *simple* factors) and the structure of finitely generated modules over a general Euclidean (or principal ideal) domain are not mentioned.

In the middle of the book are included two chapters on lattice- and set-theory, covering such notions in the former as modularity, distributivity and complementation. Boolean algebras are discussed briefly, and lattice-theoretic ideas are applied in the study of the Noetherian decomposition of ideals in a commutative ring. The set-theoretic material centers on the various devices for carrying out transfinite induction; the equivalence of Zorn's maximality principle, the axiom of choice and the well-ordering principle is established.

There are three non consecutive chapters dealing with rings and fields. Polynomial and formal power series rings are introduced, and factorization in the former is treated. The lattice-based results on intersections of irreducible ideals previously indicated are applied to obtain the corresponding results in Noetherian rings. The Hilbert basis theorem is included, but the corresponding results on power series rings and finitely generated modules are not. The final chapter of the book is the best for making many important classical notions and results conveniently available. These include root- and splitting-fields for polynomials, some results on the theory of polynomial equations over division rings, algebraic integers, structure of finite fields, theorem of the primitive element, Wedderburn's theorem on finite division rings, algebraic closure of a field (Zorn's construction), the Hilbert *Nullstellensatz*, and classical Galois theory.

Linear algebra is given short shrift; vector spaces are introduced as a special case of modules, which are a special case of groups with operators, and which incidentally give a convenient interpretation to the notion of integral dependence of an element of an extension of an

integral domain in the last chapter. No further indication of the versatility and importance of the notion of module seems to be present in the book. The particularly simple algebraic structure of vector spaces from the viewpoint of groups with operators is pointed out, followed by some introductory results on linear transformations and matrices. Canonical forms are not treated, the main result in this direction being the decomposition of a space into invariant subspaces corresponding to the distinct prime-power factors of the minimum polynomial of a given transformation. The Cayley-Hamilton theorem is proved and diagonalizable matrices are characterized in terms of their minimum polynomials.

The book is in hard covers, and the printing is clear. If there are typographical errors, they have not caught the reviewer's notice. The table of contents and index are quite complete.

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