BOOK REVIEWS

Integral operators in the theory of linear partial differential equations. By Stefan Bergman. Springer, Berlin, 1961. 8+145 pp. DM39.80.

The report summarizes the recent development in the theory of integral operators for generating solutions of linear partial differential equations from complex analytic functions. The intention of this approach is the development of parts of the theory of those equations on the basis of complex analysis, and the use of the operators may be regarded as a "translation principle" under which certain theorems on analytic functions carry over into theorems which characterize various general properties of those solutions. In this respect the operator theory seems to be a valuable addition to the classical theory which concentrates on existence and uniqueness problems. The author uses special operators which preserve properties such as the validity of series developments and the connection between the coefficients of these series and the location and character of singularities.

The consideration starts with operators $P(f)$ for generating solutions u of partial differential equations in two variables. Here $f(z)$ is an analytic function, called the associate of the solution u. Various representations of the so-called Bergman operator of the first kind are given. This operator is particularly useful for treating the coefficient problem, because it has a very simple inverse. Another important operator considered in the first chapter is the integral operator of exponential type, which generates solutions satisfying ordinary differential equations, in addition to the partial differential equation.

The next chapter is devoted to the Whittaker-Bergman operator for generating harmonic functions of three variables. Choosing the associates in a systematic fashion the author obtains a classification of the corresponding harmonic functions and discusses general properties enjoyed by all the functions of each such class.

In Chapter 3 it is shown that more general solutions of differential equations in three independent variables can be treated by operator methods. In fact, there exist operators transforming harmonic functions of three variables into those solutions. The consideration concentrates on those operators which preserve certain properties of the harmonic functions involved.

The application of the operators to systems of partial differential equations is still in its initial stage. An introduction to this part of the theory is given in Chapter 4, and it is shown that some of the operators considered before have analogues in connection with those systems.

Equations of mixed type and elliptic equations with coefficients that have singularities or are not analytic functions are considered at the end of the book.

The report includes a rather complete list of references to literature, and original papers are quoted throughout the text. Some of the proofs have been omitted in particular in cases where the transition to original papers is not difficult. Integral operators have applications in hydrodynamics and other fields, but these are not considered here.

ERWIN KREYSZIG

Theory of algebraic numbers. By E. Artin. Notes by Gerhard Würges from lectures held at the Mathematisches Institut, Göttingen, Germany in the winter semester, 1956–57. Translated and distributed by George Striker, Schildweg 12, Göttingen. 172 pp. $2.50.

Professor Artin is well known not only as a master of modern algebra, a subject to which he has made important contributions, but also as a master of elegant and original exposition. It is needless to say that the present volume based on his Göttingen lectures has all the characteristic features associated with his name though it is not quite clear what part he has taken in its production. Since there is now a tendency to drop the word modern, a better title perhaps for this enjoyable little book would be "Lectures on Algebra with Applications to Algebraic Numbers." It develops the subject as axiomatically as possible and makes clear the assumptions and axioms upon which the treatment depends. The knowledgeable reader will note that the ideas introduced will be available for the study of related topics and also applicable to other disciplines. The book will be highly valued and appreciated by those who are curious to see how the subject is developed from the apparently remote initial concepts and definitions, and how these lead to the establishment of the essential results in the theory and how they are brought into the picture. They are sure to find great pleasure in reading this book. They should not, however, be beguiled into thinking there is nothing more to be done in algebraic number theory than axiomatization. There are vast domains not touched upon here. The size of the book makes it obvious that its scope will be limited, and it must be borne in mind that the book had its origin in a one-semester course of lectures and is really of an introductory character.

Next a word of warning. There are other readers for whom Artin, it may be fairly said, did not intend to cater in his lectures, namely,