Vorlesungen über Funktionentheorie. By A. Dinghas. Springer Verlag, Berlin, 1961. 15+403 pp.

Currently the collection of textbooks and monographs that aim to present the classical theory of functions of a single complex variable and possibly aspects of its subsequent development is vast. This remark is not a complaint. On the contrary. The teacher who is concerned with the theory of functions of a complex variable has never been in a more favorable situation as far as the availability of a rich and varied collection of monographs and textbooks is concerned. This is a phenomenon of the last fifteen years or so. There were well-established texts in the period preceding World War II—such as the Lehrbücher of Osgood, Hurwitz-Courant, and Bieberbach, the Cours and Traités of Paris, the treatises of Dienes, Titchmarsh, and Saks-Zygmund (available then only in Polish but of recognized importance in spite of the linguistic barrier), the celebrated monographs of R. Nevanlinna, the Borel series, the "Aufgaben und Lehrsätze" of Pólya and Szegö, and Landau's "Neuere Ergebnisse." But it was inevitable that the burgeoning research activity in the theory of functions of a complex variable which has taken place in the last three decades should at last find itself reported in the monographic literature. In this connection we cite two treatises of major size: Golusin's *Geometric theory of functions of a complex variable* now available to a large group of readers through its translation into German, and Tsuji's *Potential theory in modern function theory*, both directed to students who have gone beyond the elements.

It is by keeping in mind the historical situation we have described that we can appraise correctly the special character of the *Vorlesungen über Funktionentheorie* of Professor A. Dinghas. Its goal is an *exposé d'ensemble* of the classical theory of functions of a single complex variable together with a number of its principal modern developments, especially those of the last three decades, beginning with first principles and advancing to the frontiers of modern investigations. Some idea of the magnitude of such an undertaking may be had when we realize that the author has not contented himself to present the classical groundwork of the subject in the old traditional manner, but has taken into consideration the reworking of the fundamentals by such mathematicians as Artin (Notre Dame Monograph) and Ahlfors (Complex Analysis). As is well known, the homology-theoretic exposition of the classical Cauchy theory given by Ahlfors has left its mark and has brought to the fore a point of view that no writer of a modern text on the theory of functions of a complex variable can possibly overlook, adopt it or not as he sees fit for his special purposes. Although the book is distinguished by its comprehensiveness and for that reason serious account has been taken of the reworkings of foundational material, it is especially in the treatment of the topics not customarily presented in the standard one or two semester introductory courses on complex function theory that there is to be found, in my opinion, the special flavor of the book. The topics of this character are rich in variety. In part their treatment is incorporated in the text and in part in the "Ergänzungen und Aufgaben" appended to each chapter which run the gamut from straightforward exercises to statements of major results with indications of proof and bibliographical references.

From the point of view of American instruction, the material of the appendices would be excellent either for independent study by an advanced student interested in getting a picture of modern developments or for a proseminar in which the "Ergänzungen und Aufgaben" would be worked and discussed. Another useful element of the book is the inclusion in most chapters of historical accounts of the material. Here regard is given to recent work.

Among the special features of the book the following deserve mention. The gap theorem of Fabry is developed with the aid of Turán's Lemma (Hungarica Acta Math., 1947). The chapter on the gamma and zeta functions includes the Hadamard-de la Vallée Poussin theorem and Wiener's proof of the prime number theorem. In the chapter on majorization and growth problems the fecund method of Carleman (C. R. de l'Acad des Sci. de Paris 1933) plays a central role. The final chapter includes the development of the Nevanlinna theory for meromorphic functions in the neighborhood of an isolated singularity.

The author regrets the brevity with which such topics as algebraic functions, Riemann surfaces, uniformization and elliptic functions are treated. However, the references are completely up-to-date and extensive. One topic that might well deserve a full treatment in a text of the present character is the theorem of Osgood-Taylor-Carathéodory concerning the boundary behavior of conformal maps of Jordan regions. Only a statement with bibliographical and historical indications is given.

The contents of the book follow. The first three chapters treat background material: the complex plane, its elementary topological properties, local properties of analytic functions. More than one heretic will be pleased to see that the exposition is not burdened by elaborate accounts of elementary functions—material fittingly relegated to exercises at this level of a student's development. The fourth chapter treats the principal theorems of the Cauchy theory together with its applications. Here the residue calculus is studied. Account is given of the homology-theoretic formulation of the Cauchy theory.

The second part of the book turns to the Riemann-Weierstrass approach. Chapter five is concerned with representation theorems, analytic continuation and noncontinuable power series. The sixth is dedicated to the gamma and zeta functions and the prime number theorem.

The final part of the book is entitled "Maximum principle and distribution of values." Chapter seven treats well-known majorization and growth problems: the Schwarz Lemma, the three circles theorem, the Phragmén-Lindelöf-Nevanlinna theorems, the Wiman theorem, the Denjoy-Carleman-Ahlfors theorem. The last two chapters are quite extensive. Chapter eight, Geometric function theory and conformal mapping, takes up hyperbolic geometry, the Riemann mapping theorem, the Dirichlet problem, the Evans-Selberg theorem, the Picard theorem and cognate results, distortion theorems. The supplementary section of the chapter includes an extensive summary of van der Waerden's treatment of uniformization. Chapter nine is devoted to the Nevanlinna theory.

Typical for the up-to-date character of the references are the last two of the text: Matsumoto, J. Sci. Hiroshima Univ. (A) **24** (1960) and Carleson, Bull. Amer. Math. Soc. **67** (1961).

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Differential geometrie. By Detlef Laugwitz. B. G. Teubner Verlagsgesellschaft, Stuttgart, 1960. 183 pp. DM 24.60.

This is intended to be an introduction to classical differential geometry, tensor calculus, and Riemannian geometry. It is that and much more. The amount of material packed into 180 pages is amazing. Yet the book is also readable, with many passages giving motivation and surveying the methods.

The chapters are: I. LOCAL DIFFERENTIAL GEOMETRY OF SPACE CURVES (12 pages). II. LOCAL DIFFERENTIAL GEOMETRY OF SURFACES (43 pages). III. TENSOR CALCULUS AND RIEMANNIAN GEOMETRY (43 pages). IV. FURTHER DEVELOPMENT AND APPLICATIONS OF RIEMANNIAN GEOMETRY (49 pages). V. TOPICS IN DIFFERENTIAL GEOMETRY IN THE LARGE (17 pages).

Besides most of the standard material one would expect to find under such headings, many topics not usually covered in a text at this level appear: holonomy group of an affine connection; the kine-