

analogous fashion but lead to very messy formulae, which furthermore give no additional stable information.

4. Finally a word concerning the proof of Theorem I. It is a known result that when $X = \text{point}$, then $KO\{\mathbf{S}(E)\} = KO(S^{8n})$ is generated by 1 and y . (See [2]). Hence (2.1) proves the first statement of Theorem I whenever E is trivial. Now an inductive Meyer-Vietoris argument yields the general case.

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HARVARD UNIVERSITY

A CONNECTION BETWEEN TAUBERIAN THEOREMS AND NORMAL FUNCTIONS

BY G. T. CARGO¹

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The purpose of this note is to point out that certain Tauberian theorems follow immediately from some recent research of Lehto-Virtanen and Bagemihl-Seidel.

Let D denote the open unit disk, let C denote the unit circumference, and let $\rho(z_1, z_2)$ denote the non-Euclidean hyperbolic distance between the points z_1 and z_2 in D .

THEOREM. *Suppose that $f(z) = \sum a_n z^n$ and that $n|a_n| \leq M$ ($n = 1, 2, \dots$) for some constant M . Further, suppose that $\{z_n\}$ is a sequence of points in D converging to a point ζ in C with the property that $\rho(z_n, z_{n+1}) \rightarrow 0$ as $n \rightarrow \infty$. Then, if $f(z_n) \rightarrow c$ as $n \rightarrow \infty$, the series $\sum a_n \zeta^n$ converges to the sum c .*

PROOF. The hypothesis implies that $|f'(z)| \leq M/(1 - |z|)$. Consequently, $\rho(f(z))|dz| \leq 2M d\sigma(z)$ holds for all z in D where $\rho(f(z)) = \frac{|f'(z)|}{(1 + |f(z)|^2)}$ denotes the spherical derivative of f and $d\sigma(z)$

¹ National Academy of Sciences-National Research Council Postdoctoral Research Associate on leave from Syracuse University.

$= |dz|/(1 - |z|^2)$ denotes the hyperbolic element of length. From this we infer at once (see [2, Theorem 3]) that f is normal in the sense of Lehto and Virtanen; and, invoking a theorem of Bagemihl and Seidel [1, Theorem 2], we conclude that f has the angular limit c at ζ . The theorem now follows from Littlewood's Tauberian theorem for radial approach.

The theorem contains the Hardy-Littlewood Tauberian theorem for curvilinear approach as a special case.

It is now obvious that one can formulate and prove a number of Tauberian theorems by making use of various known properties of normal functions. Conversely, known Tauberian theorems will sometimes suggest properties of normal functions. For example, the fact that a holomorphic function in D having a finite Dirichlet integral is normal yields at once an extension of a familiar Tauberian theorem.

The author will discuss these matters in more detail elsewhere.

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NATIONAL BUREAU OF STANDARDS