RESEARCH PROBLEMS

8. Albert A. Mullin: *Recursive function theory.* (*A modern look at a Euclidean idea.*)

Consider Euclid's elementary scheme for proving the infinitude of the primes (*Elements*, Book IX, Proposition 20). Using that idea as a paradigm, and with N the set of natural numbers, generate the following recursively enumerable (in the sense of, e.g., E. L. Post, Bull. Amer. Math. Soc. **50** (1944), 284–316) set $\{p_i: i \in N\}$ of primes: $p_1 = 2$, $p_2 =$ (*least* prime factor of p_1+1) $= 3$, $p_3 =$ (least prime factor of $p_1 \cdot p_2 + 1$) $= 7$, $p_4 =$ (least prime factor of $p_1 \cdot p_2 \cdot p_3 + 1$), $\cdots$, p_{n+1} $=$ (least prime factor of $p_1 \cdot p_2 \cdots p_n + 1$), $\cdots$. Clearly $\{p_i: i \in N\}$ is infinite, since by an analogue to Euclid's argument $\{p_{i+1}\} \cap \{\bigcup_{k=1}^{i} p_k\} = \varnothing$ for every $i \in N$; further the "computer" process never generates any prime *twice.* In addition the enumeration is *not* one that is in increasing order (e.g., $p_4 = 43$, but $p_5 = 13$). Hence by a result of S. C. Kleene $\{p_i: i \in N\}$ may not be recursive. (i) Is $\{p_i: i \in N\}$ recursive? (ii) Does $\{p_i: i \in N\}$ generate all primes? If (i) is answered negatively, then (ii) would be answered negatively. Now generate the (infinite) recursively enumerable set $\{q_i: i \in N\}$ obtained as in the above effective (e.g., computer) process by replacing "least prime factor" by "greatest prime factor." As before no prime is generated twice. Unlike the first five elements of $\{p_i: i \in N\}$, the first five elements of $\{q_i: i \in N\}$, viz., $q_1 = 2$, $q_2 = 3$, $q_3 = 7$, $q_4 = 43$, $q_5 = 139$, *are* in increasing order. (iii) Does the above process generate $\{q_i: i \in N\}$ in increasing order, and hence make it an infinite recursive proper subset of the set of all primes? (iv) Is $\{q_i: i \in N\}$ still recursive even though (iii) may be answered negatively? (v) If (iii) is answered negatively, does $\{q_i: i \in N\}$ generate all primes? Clearly if (iv) is answered negatively, then so is (v) answered negatively.

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9. A. V. Balakrishnan: *Geometry.*

In the class of all polytopes with N vertices whose circumsphere is the unit sphere in E_m [Euclidean space of dimension m] find the one that has maximum mean-width.

The mean-width of a convex body is defined as:

$$\frac{2\int_S H(e)\,dS}{\int_S dS}$$

where $H(\cdot)$ is the support function for the body, "e" denotes unit vector, and dS is the surface element of the unit ball (sphere) in E_m.

Answers are known in two extreme cases: for $m=N-1$, it is the regular simplex, and for $m=2$, it is the regular polygon.

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10. O. Taussky: *Matrix commutators of higher order.*

Let A, B be $n\times n$ matrices over an algebraically closed field. It is known that if B commutes with any matrix X which commutes with A then B must be a polynomial in A with coefficients in the same field (see J. H. M. Wedderburn, *Lectures on matrices*, Amer. Math. Soc. Colloq. Publ. Vol. 17, 1934). Recently M. Marcus and N. A. Khan (Canad. J. Math. **12** (1960), 259–277) showed the same result under the assumption that X commutes with $XB-BX$ whenever A commutes with $AX-XA$. Even more recently M. F. Smiley (Canad. J. Math. **13** (1961), 353–355) generalized this result to commutativity of arbitrary orders allowing the characteristic to be zero or at least n.

Is Smiley's result true for not algebraically closed fields?

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