$i=1, \cdots, c$, and $c \leqq 2$, r_1 or $r_2=1$. But this implies that A is a permutation matrix.

CONJECTURE. *If $A=(a_{ij})$ is an n-square* (0, 1)*-matrix then*

$$p(A) \leqq \prod_{i=1}^{n} (r_i!)^{1/r_i} \tag{3}$$

with equality if and only if there exist permutation matrices P and Q such that PAQ is a direct sum of matrices all of whose entries are 1.

The conjecture is known to be true for all (0, 1)-matrices whose row sums do not exceed 6.

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THE COLLINEATION GROUPS OF DIVISION RING PLANES. I. JORDAN ALGEBRAS

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In this note, we outline a method which reduces the determination of the collineation group of a division ring plane to the solution of certain algebraic problems—in particular, to the question of when two rings of a certain type are isomorphic. This method is then applied to planes coordinatized by finite dimensional Jordan algebras of characteristic $\neq 2, 3$, and their collineation groups are determined. Complete arguments and detailed proofs will appear elsewhere.

1. Let $\mathfrak{R}$ be a nonalternative division ring, let $\pi(\mathfrak{R})$ be the projective plane coordinatized by $\mathfrak{R}$, and let $G(\pi)$ be the collineation group of π. Then (see [1]) $G(\pi)$ possesses a solvable normal subgroup whose structure is known, the elementary subgroup, such that the factor group is isomorphic with the group of *autotopisms* of $\mathfrak{R}$, $A(\mathfrak{R})$. Also, $A(\mathfrak{R}) \approx H(\pi)$, where $H(\pi)$ consists of those elements of $G(\pi)$ which leave fixed the points (∞), (0), and (0, 0). (See [2], Chapter 20 for the coordinatization of projective planes.)

Let $B(\mathfrak{R})$ be the *automorphism* group of $\mathfrak{R}$. Then $B(\mathfrak{R}) \approx H_1(\pi)$, where $H_1(\pi)$ consists of those elements of $H_1(\pi)$ which leave the point

(1, 1) fixed. Thus, a coset decomposition

$$H(\pi) = \sum_i H_1(\pi)\alpha_i \tag{1}$$

can be obtained, and our first result, which is easily proved, is

THEOREM 1. *ϕ_1, ϕ_2 are in the same coset if and only if* $(1, 1)\phi_1 = (1, 1)\phi_2$.

Now, call a pair (a, b) *admissible* if there is an element of $H(\pi)$, α, such that $(1, 1)\alpha = (a, b)$. If all admissible pairs can be determined, then we will know what each coset does to the point (1, 1), and can actually begin to look for coset representatives.

At this time, we need

THEOREM 2. *Let $\mathfrak{R}$, $\mathfrak{R}'$ be the two coordinate rings for a plane defined by the quadrangles (∞), (0), $(0, 0)$, $(1, 1)$ and $(\infty)'$, $(0)'$, $(0, 0)'$, $(1, 1)'$, respectively. Then $\mathfrak{R}$ and $\mathfrak{R}'$ are isomorphic if and only if there is a collineation α such that $(\infty)\alpha = (\infty)'$, $(0)\alpha = (0)'$, $(0, 0)\alpha = (0, 0)'$, and $(1, 1)\alpha = (1, 1)'$.*

We now recoordinatize $\pi(\mathfrak{R})$ using the quadrangle $(\infty)' = (\infty)$, $(0)' = (0)$, $(0, 0)' = (0, 0)$, $(1, 1)' = (a, b)$. Call the new coordinate ring $\mathfrak{S}_{a,b}$. Then $\mathfrak{R}$ and $\mathfrak{S}_{a,b}$ are isotopic, and Theorem 2 says that (a, b) is an admissible pair if and only if $\mathfrak{R}$ and $\mathfrak{S}_{a,b}$ are isomorphic.

Upon recoordinatizing, we find that $(\mathfrak{R}, +)$ and $(\mathfrak{S}_{a,b}, +)$ are isomorphic under the trivial identification of elements, and that multiplication in $\mathfrak{S}_{a,b}$ can be defined by

$$x * y = \left\{ (xR_{a^{-1}}^{-1}) \left[((yR_{a^{-1}}^{-1})(bL_a^{-1}))L_a^{-1} \right] \right\} R_{bL_a^{-1}}^{-1} R_{a^{-1}}, \tag{2}$$

where R_x and L_x represent right and left multiplication in $\mathfrak{R}$.

2. In trying to find all admissible pairs when $\mathfrak{R}$ is a finite dimensional Jordan algebra, one needs to prove the following theorem which is an important tool in the subsequent analysis.

THEOREM 3. *The left, middle, and right nuclei of a finite dimensional Jordan division algebra are all equal.*

The next step is fairly long and difficult, and consists in using in various subtle ways the assumptions that $\mathfrak{S}_{a,b}$ is commutative and satisfies the Jordan identity

$$R_x R_{x^2} = R_{x^2} R_x \tag{3}$$

until the following result is reached:

THEOREM 4. *If $\mathfrak{R}$ is a finite dimensional Jordan algebra of characteristic $\neq 2, 3$, then (a, b) is an admissible pair if and only if a and b are both elements of the center of $\mathfrak{R}$.*

Thus, we know not only that $\mathfrak{S}_{a,b}$ and $\mathfrak{R}$ are isomorphic if and only if a and b are in the center of $\mathfrak{R}$, but from (2), we see that

$$x * y = xy, \tag{4}$$

which says that the trivial mapping is an isomorphism. But actually knowing an isomorphism between $\mathfrak{R}$ and $\mathfrak{S}_{a,b}$ allows one to write explicitly a set of coset representatives, $\alpha_{a,b}$, for (1). These coset representatives are defined by:

$$\begin{aligned}(x, y)\alpha_{a,b} &= (xa, yb)\\ (m)\alpha_{a,b} &= (ma^{-1}b).\end{aligned} \tag{5}$$

All that remains, then, is to determine the automorphism groups of these Jordan algebras. But this has been done for most of the classes of such algebras, and for a complete account of what is known about the automorphism groups of Jordan algebras, see [**3**, pp. 190–191].

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