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BOOK REVIEW

Mathematical thought from ancient to modern times, by Morris Kline. Oxford University Press, New York, 1972. xvii+1238 pp.

The history of mathematics is an enticing but neglected field.

One reason for this situation lies in the nature of intellectual history. For any theoretical subject x, telling the story of x is not a conceptually distinct undertaking from describing the theory of x, though the two presentations often appear in different guises. The readership of a serious history of x will thus be largely limited to the few specialists in x, a small circulation at best. Worse yet, mathematics, or science for that matter, does not admit a history in the same sense as philosophy or literature do. An obsolete piece of mathematics is dead to all but the collector of relics. Discovering that the Babylonians knew harmonic analysis may be an astonishing feat of scholarship, but it is a supremely irrelevant piece of information to working scientists. Few of the serious historians of mathematics have realized this; as a result, we are saddled today with competent histories of Greek and Renaissance mathematics, but we sadly lack such items of burning interest as "The Golden Days of Set Theory (1930–1965)" "Topology in the Age of Lefschetz (1924–1953)", "The Beginnings of Probability (1932–$\cdots$)", to cite but a few possible titles.

Faced with these and many other problems, Morris Kline has chosen the courageous avenue of compromise. In his book, the Greeks get a 15% cut, the Egyptians are whisked on and offstage, the Arabs and Renaissance together make a fleeting 10% appearance, and the drama begins with René Descartes on Chapter 15 out of 51.

Synthetic geometry is Morris Kline's first love, and he returns to it with predictable regularity. It is thus no wonder that his treatment of Greek geometry succeeds in being readable, at times downright interesting. Ancient mathematics has a way of appearing to us coated with a dusty layer of weirdness, which historians have occasionally attempted to brush off with a phony emphasis on the human angle, as if we could reconstruct what made the Greeks tick without the aid of science fiction. Kline's solution, instead, is to translate into contemporary language and values the best of Greek geometry, with brief mention of the authors. His is a history of ideas, not one of men. Nonetheless, one is pleased to find throughout the book a scrupulous fairness in giving the little guy his due, and in emphasizing the greatness of some of the lesser known names. For example, Apollonius gets equal billing with Euclid, as he should.

To pick another example at random, in the farewell chapter to synthetic geometry, the names of Pieri and Veronese are mentioned together with. that of Hilbert, though not on a par; but then, Hilbert had the glory of Göttingen behind him, while Pieri—whose treatment of the foundations of geometry Russell considered superior—was sweating at Parma U. The book abounds with such touching episodes, though a really thorough straightening of mathematical priorities is not carried out; it might have made quite a shocker.

The climax comes in the chapters on nineteenth century analysis Kline's expertise in the subject makes his presentation a delight. His device of combining historical development with a clear, linear description of the material turns the book into a useful *précis* of mathematics up to the turn of the Century. The lucid chapter on asymptotic series—inexplicably combined with the obsolete notion of summability—makes one wonder why such an obviously fundamental topic has not found its way into the standard calculus curriculum. Similarly, his two-page presentation of Fredholm's ideas on integral equations leaves one filled with wonder. On the other hand, the two chapters on differential geometry might have gained by the use of contemporary notation; in their present form, they will be all but inaccessible to students trained in the last fifteen years. But Kline has a bone to pick with certain contemporary mathematical schools. One may wonder why the chapters on functional analysis and topology are also presented with a decidedly conservative slant, which is at variance with present-day practice in graduate courses, and does not relate to the current problematic of these subjects. Algebra and logic, the two triumphant if not always amicable contenders for centerstage in present-day pure mathematics, get only a fleeting mention. There is not a word on group representations, classical as the subject is, and the deeper results in number theory are hardly touched upon, perhaps because these subjects call for higher mathematical sophistication, and Kline wants to keep his requirements down to the second-year calculus level.

Morris Kline's abrupt stop at the nineteenth century, as well as his recurring emphasis on synthetic geometry, are not simply a whim. This is not only a handbook of basic mathematical culture, nor merely a good reference book of mathematical history. It is also meant as a message; it stresses and at times betrays a forceful position on some controversial issues in contemporary mathematics. A grasp of these issues is a prerequisite for the understanding of the book's unity of purpose. Briefly, these issues are: (1) the originality of contemporary mathematics, and (2) the raging battle over the "New Math" reform in the high schools.

The first accounts for the stress on the remote past. It is the author's contention that contemporary mathematics runs a poor second to the

mathematics of the past century. The excesses of abstraction and notation of the present conceal dearth of ideas and lack of novelty, or so the author claims. For Kline, mathematics is primarily a body of facts; proof is only the tool for their confirmation. Some contemporary mathematicians think otherwise: for them, according to Kline, the results of mathematics are contingent upon and secondary to the formal-deductive structure of mathematical reasoning.

There are strong points for both sides. Kline's book is in part an implicit argument in support of the author's thesis. It is a showcase display of the great facts of mathematics, presented in their shining light and eternal value. On leafing through the book, one is struck how hard indeed it is to find in our century new mathematical theories as obviously useful as, say, the calculus of variations (Chapters 24 and 30) or the oscillation theory for solutions of differential equations (Chapter 29). (Strangely, in physics the opposite seems to be true.) However, Kline's assertion that "abstract algebra has subverted its own role in mathematics" (p. 1157) is at best a partisan one. If we are to admire Descartes for his reduction of geometry to algebra (Chapter 15), then why is contemporary (commutative) algebra not equally admirable for successfully unifying and cross-fertilizing such disparate undertakings as number theory, algebraic geometry, differential geometry and topology? Present-day mathematics is poorer in old-fashioned facts and more abundant in concepts, but its avowed aim is a synthesis and simplification which may bring about far more sweeping changes than a simple accumulation of facts would. Ours is the age of "terrible simplifiers" forecast by Jakob Burckhardt, whose effects upon the future may still be in the realm of conjecture, but will not in any case be negligible.

On the issue of the "New Math", Morris Kline is a paladin of the Old Order. By wiping synthetic geometry out of the curriculum and replacing it with subjects that make fewer demands for the student's active participation, this reform is threatening to turn math from a "hard" into a "soft" subject. The old emphasis on math as a collection of facts about the world has been replaced by a stress on deductive argument and formal thought. An increasing number of mathematicians are coming to doubt the value of this reform; Kline's book, intended to show teachers and other practitioners "what good math used to be like", will be a rallying point for the opposition.

It is easy to find something to criticize in a treatise 1200 pages long and packed with information. But whatever we say for or against it, we had better treasure this book on our shelf, for as far as mathematical history goes, it is the best we have.

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