Hasse's work on orders in complete skewfields is presented. An interlude on Morita equivalence permits passage to the case of maximal orders in separable algebras over local fields. Classical techniques then transfer the information thus far derived to the case of maximal orders over arbitrary Dedekind rings. At each stage, ideal theory, different, discriminant, and norms are discussed in full detail.

The latter part of the book is devoted to some interesting special topics in the theory of orders. The theory of Brauer groups is developed from the point of view of crossed products, and, in particular, of cyclic algebras. This discussion is very thorough, and would provide all the background necessary for the applications to classfield theory. The only important results asserted without proof are the Hasse Norm Theorem and the Grundwald-Wang Theorem.

Other topics covered are Eichler's Theorem on reduced norms, hereditary orders and some of the recent results of the author, Ullom, Frölich, and others on class groups and Picard groups of orders.

The book has been written with great care, and is a pleasure to read. Unlike many books at such an advanced level, it contains many interesting exercises, with hints where appropriate; it contains almost no misprints or mistakes. It is essential to the library of every working algebraist and number theorist.

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The Stone-Čech compactification, by Russell C. Walker, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 83, Springer-Verlag, New York, 1974, x + 332 pp., $30.40.

Stone and Čech published their papers on βX, the "Stone-Čech compactification" of X, in 1937. Here X is a completely regular Hausdorff space. The space βX is characterized as the maximal compactification of X: every mapping from X to a compact space K extends to a mapping from βX to K. It is sufficient to state this for $K \subset \mathbf{R}$: βX is that compactification of X in which X is C^*-embedded, i.e., every bounded continuous real-valued function on X extends to βX.

With the existence and uniqueness of βX thus established, from one point of view the subject is closed. From another point of view it has just been opened. What are particular properties of βX and how do they reflect properties of X? Čech raised some specific questions, and answered some of them, in his original paper. For instance, βX is connected if and only if X is connected; on the other hand, βX is *never* metrizable (for noncompact X). What about X^*, i.e., $\beta X \setminus X$? What does $\mathbf{N}^*$, for example, "really look like?" What is its cardinal number, for that matter? Pospíšil answered this last question in a note published side by side with Čech's paper: for any infinite discrete space D, $|\beta D| = \exp\exp|D|$ and hence $|D^*| = \exp\exp|D|$; in particular, $|\mathbf{N}^*| = 2^{\mathfrak{c}}$. (The year before, Hausdorff had solved the same problem but in a form not recognized at the time as equivalent.)

The next major development was the 1939 paper of Gelfand and Kolmogoroff, in which attention was called to the ring $C(X)$ of all continuous functions from X to $\mathbf{R}$ (bounded or not) and which announced the result that the maximal ideals in $C(X)$ correspond to the points of βX. The like result about maximal ideals in $C^*(X)$ (the subring of bounded functions in $C(X)$) was natural enough and had been noted by Stone, but the result for $C(X)$ was a surprise.

There was a lull until 1948 when Hewitt's pathbreaking paper appeared. This paper inspired a wealth of research during the fifties about $C(X)$, $C^*(X)$, and βX. Henriksen and the reviewer obtained relations between the topology of X and the ring structure of $C(X)$ (e.g., "P-spaces" and "F-spaces"). W. Rudin came up with the surprising discovery (with the help of the continuum hypothesis) that $\mathbf{N}^*$ is not homogeneous. The appearance of the book **[GJ]** in 1960, containing these and other major results, stimulated still further research about βX.

The book under review is a sequel to **[GJ]**, devoted to presenting the results of that research. It represents the author's dissertation for the Doctor of Arts. It is a first-class piece of organization, synthesis, and exposition. The writing is concise and clear. Proofs are included except where they would take the reader too far afield; thus the book is essentially self-contained. It is accessible to the student who has had a solid introduction to topology. Chapters 1 and 2 present preliminary material about βX and Boolean algebras, and the concluding Chapter 10 discusses βX from a categorical point of view. (This last appears to be tacked on.) The main topics in the book proper, Chapters 3–9, include C^*-embedding of subspaces, characterizations of $\mathbf{N}^*$, Frolík's theory of types of ultrafilters and nonhomogeneity of X^* (without the continuum hypothesis), cellularity and density of X^*, fixed points of mappings, product theorems, connectedness. There are a comprehensive bibliography and index, plus a helpful author index. Each chapter closes with a set of exercises giving extensions of the theory along with additional references to the literature. I plan to teach from the book next year.

I detected only a half-dozen misprints or related slips, all minor. E.g., "Glicksberg" and "Gordon" are misspelled two or three times; the reference on p. 216 to Proposition 1.65 should be to 1.66; the index entry "relatively compact" refers only to the definition on p. 236, whereas the term is defined and used on p. 124.

I have a more important criticism to register–against the publishers, for whom I refereed the manuscript. For some reason, they failed to transmit my comments to the author. Some of these comments concern terminology. For one thing, I recommend containment of the tendency to use "containment" to signify set-theoretic inclusion. Second, while it is a good thing to have a term for $\beta X \setminus X$, I think the appropriate notion is one of complement in βX rather than of growth of X; in any case, the author frequently falls into the trap of calling it the growth of βX (which is technically ambiguous, as βX need not determine X) and on at least one occasion (p. 226, lines 6–7) uses both forms in the same sentence.

Next, there are some faulty credits, apparently caused by too hasty reading of the sources. Corollary 1.25 is due not to Stone but to Gelfand and Kolmogoroff. Proposition 1.66, credited to Curtis alone, was discovered independently by Henriksen, as Curtis points out. Proposition 3.30, credited to me alone, is joint work with Fine, as is stated explicitly in the expository paper from which the proof is taken. Contrary to the impression given in 4.43, Fine and I obtained a dense set of remote points, not just one. Proposition 7.2, attributed to me, is due to Comfort and Negrepontis.

Finally, I have some comments about a couple of proofs. It seems heavy-handed to prove that a countable completely regular space is normal by arguing (pp. 57 and 71) that it is regular and Lindelöf, hence paracompact, hence normal–as a simple direct proof is available [**GJ**, 3B.4,5 and, more generally, 3D.4]. Next, the author should note that the result obtained in 5.21, namely, that $\beta\mathbf{N}$ is the unique extremally disconnected compactification of $\mathbf{N}$, is an immediate corollary of problem 2J(4). Finally, it is a pity to omit a proof that X^* is an F-space for locally compact and σ-compact X on the grounds that the proof in [**GJ**] is algebraic (p. 36); the original, long proof was not algebraic, and, anyhow, Negrepontis came up with a short one [Proc. Amer. Math. Soc. **18** (1967), 691–694]: to show that a cozero-set A in X^* is C^*-embedded, note that since X is locally compact X is open in βX (and in $X \cup A$), and hence X^* is compact whence A is σ-compact; since X is σ-compact, $X \cup A$ is σ-compact and hence normal; consequently, A is closed in the normal space $X \cup A$ and is therefore C^*-embedded in $X \cup A$, hence in $\beta(X \cup A) = \beta X$, hence in X^*.

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Variation totale d'une fonction, by Michel Bruneau, Lecture Notes in Mathematics, vol. 413, Springer-Verlag, Berlin, 1974, 332 + xiv pp., $12.30.

Real analysis was an active research area at the beginning of the twentieth century when mathematicians were exploring the implications of Lebesgue theory. The recent appearance of H. Federer's extraordinary book, *Geometric measure theory*, shows that there is continuing interest in this field.

The book under review is an account of recent research on variations and measures associated with functions of a real variable. Since one of these, the Wiener pth power variation, is a little known concept with applications in various branches of analysis, it may be worthwhile to list some facts about it here.