diminishing returns. It is probably hopeless to buck the current trend toward eliminating all requirements for foreign languages, and we can probably no longer ask that advanced graduate students have minimal competence in simple, elegant French. But we can perhaps ask that the translations serve the students as well as the original.

REFERENCES

1. J. L. Doob, *Stochastic processes*, Wiley, New York; Chapman and Hall, London, 1953. MR **15**, 445.

2. M. Loève, *Probability theory*, 3rd ed., Van Nostrand, Princeton, N.J., 1963. MR **34** #3596.

3. L. Breiman, *Probability*, Addison-Wesley, Reading, Mass., 1968. MR **37** #4841.

4. J. Neveu, *Bases mathématiques du calcul des probabilités*, Masson, Paris, 1964. MR **33** #6659.

5. W. Feller, *An introduction to probability theory and its applications*, vol. II, Wiley, New York, 1966. MR **35** #1048.

6. S. Karlin, *A first course in stochastic processes*, Academic Press, New York, 1966. MR **34** #8466.

7. E. Parzen, *Stochastic processes*, Holden-Day, San Francisco, Calif., 1962. MR **25** #2628.

8. B. V. Gnedenko, *Course in the theory of probability*, GITTL, Moscow, 1950; English transl. by B. D. Seckler, Chelsea, New York, 1962. MR **13**, 565; **36** #912; #913.

9. K. L. Chung, *A course in probability theory*, Harcourt, Brace and World, New York, 1968. MR **37** #4842.

10. M. Rosenblatt, *Random processes*, Oxford Univ. Press, New York, 1962. MR **24** #A3686.

11. A. Rényi, *Probability theory*, Tankönyvkiado, Budapest, 1954; English transl., North-Holland, Amsterdam; American Elsevier, New York, 1970. MR **17**, 46; **47** #4296.

12. Ju. V. Prohorov and Ju. A. Rozanov, *Probability theory*, 2nd rev. ed., "Nauka", Moscow, 1973; English transl., Springer-Verlag, New York, 1974. MR **36** #2175; **50** #3280.

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Potential theory on locally compact abelian groups, by Christian Berg and Gunnar Forst, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 87, Springer-Verlag, Berlin, Heidelberg, New York, 1975, vi + 197 pp., $25.40.

There are not many books on the general potential theory from a nonprobabilistic point of view, and nearly none is concerned with convolution kernels different from the newtonian kernel (Landkoff's book being partly an exception). Therefore this book fills a gap and will be welcome and useful.

The authors consider only a simple and important case, where everything runs smoothly: the potential kernel is a *positive* convolution operator on a locally compact *abelian* group, and is the "vague" integral of a *transient* semigroup of positive measures. There is no mention of important and recent papers on nonabelian groups and recurrent semigroups. On the other hand probabilistic interpretations in terms of Hunt's processes are not given.

A good deal of the treated material has been well known for many years, but appears for the first time in a text-book (of course such a book should have been written before). The three following topics deserve a particular mention:

(a) *The study of negative definite functions* (the terminology is due to Beurling, who has investigated these functions, mainly in the symmetrical case, but this notion is older and goes back to Von Neumann, Schoenberg, Lévy and others). Let us recall that a family $\{\mu_t\}_{t\geqslant 0}$ of probability measures on a locally compact abelian group G is a convolution semigroup if and only if there exists a continuous negative definite function ψ on the dual group $\hat{G}$ satisfying $\hat{\mu}_t = \exp(-t\psi)$ for $t \geqslant 0$, $\hat{\mu}_t$ being the Fourier transform of μ_t. The semigroup is said to be transient if the vague integral $\kappa = \int_0^\infty \mu_t\, dt$ exists (such a measure κ will be called a transient convolution potential kernel). The Port and Stone theorem states that $\{\mu_t\}_{t\geqslant 0}$ is transient if and only if $\mathrm{Re}(1/\psi)$ is locally integrable on $\hat{G}$. One gives an "analytic" proof of the "only if" part of this theorem. A parallel study of Bernstein functions leads to another representation for transient convolution semigroups of measures carried by $[0, +\infty[$ (by means of Laplace transform), and to a notion of subordinate semigroup.

(b) *The study of some remarkable convex cones of completely monotonic functions*, recently introduced by Hirsch, for instance the cone of Stieltjes transforms of positive measures carried by $[0, +\infty[$. These cones play an important role in the so called symbolic calculus on potential operators, and lead to several extensions of a theorem of Ito: if κ is a transient convolution potential kernel, and if μ is a positive measure on $[0, 1[$, then $\int_0^1 \kappa^\alpha\, d\mu(\alpha)$ is still a transient convolution potential kernel (this theorem is noteworthy, since the cone of all the transient convolution kernels on a given group is generally *not* convex).

(c) *The "rough"(not fine) potential theory with respect to a transient convolution potential kernel*: excessive measures, F. Riesz decomposition theorem, balayage theorem, identity between transient convolution kernels and "perfect" kernels (i.e. kernels associated to a "fundamental family", a notion I introduced many years ago), etc.

The book is divided into three unequal chapters. The first one (harmonic analysis, pp. 1–38) gives the definitions and statements concerning the convolution of measures on a locally compact abelian group and the rudiments of Fourier analysis on such a group (an original exhibition of the Fourier transform of a–not necessarily bounded–positive definite measure is given). Chapter two (negative definite functions and semigroups, pp. 39–96) studies the relations between these two notions, and introduces the concepts of potential operators and of subordinate semigroups. Chapter three (potential theory for transient convolution semigroups, pp. 97–190) is the longest and gives many properties of a transient convolution potential kernel.

Thus the authors are particularly interested in methods of Fourier analysis in potential theory, hence their systematic use of negative definite functions. These methods are very elegant but they apply only to convolution kernels on an abelian group. In my opinion the notion of codissipativity, due to Hirsch, might have been introduced: this notion is useful, not only for the sake of natural generalizations (for instance to convolution kernels on homogeneous spaces), but even in the abelian case. On the other hand, I could regret the lack of a complete proof for some fundamental results of this abelian theory. For instance the Lévy-Khintchin representation formula for negative definite functions is proved only in the symmetrical case (following Harzallah's method). There are other examples. And it is still a challenge for an analyst to discover a nonprobabilistic proof of the most difficult part of the Port and Stone theorem in the unsymmetrical case (the real case is much simpler and had been solved before by Beurling and myself).

The book is clear. Each of the 18 paragraphs starts with a short outline and finishes with a sufficient, but not exhaustive, bibliography. It is written with great care and there are very few misprints. Its reading is easy and enjoyable. Some straightforward proofs are left as exercises to the reader, even when the corresponding results are subsequently used. Several simple and illuminating examples are thoroughly detailed. To sum up: a highly recommendable introduction to the general potential theory from an "analytic" point of view.

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Stochastic differential equations and applications, Vol. 1, by Avner Friedman, Academic Press, New York, San Francisco, London, 1975, xiii + 231 pp., \$24.50. Vol. 2, by Avner Friedman, Academic Press, New York, San Francisco, London, 1976, xiii + 299 pp., \$32.50.

A diffusion process with values in R^d defined for some interval $[0, T]$ of time is a Markov process with R^d for its state space which has almost surely continuous trajectories. The conditional distribution of an infinitesimal increment $x(t + h) - x(t)$ of such a process given the past history $\{x(s)\}$ for $0 \leqslant s \leqslant t$ is supposed to be approximately Gaussian with mean $hb(t, x(t))$ and covariance $ha(t, x(t))$. For each t and x, $b(t, x)$ is a vector with components $\{b_j(t, x)\}$ and $a(t, x)$ is a symmetric positive semidefinite matrix with entries $\{a_{ij}(t, x)\}$. Although such a description may not hold for every diffusion process, one can describe a wide class of such processes in terms of their associated coefficients $\{a_{ij}(t, x)\}$ and $\{b_j(t, x)\}$. These are often referred to as diffusion and drift coefficients.

The problem then is to start with some given $\{a_{ij}(t, x)\}$ and $\{b_j(t, x)\}$ and then show that under suitable regularity conditions on a and b there corresponds to it a unique diffusion process. Since a Markov process is fully determined by its transition probabilities it is enough to construct the transition probability function $p(s, x, t, dy)$ from the coefficients a and b. One way of doing this is to look at some associated partial differential equations known as Kolmogorov's backward equations.

Let us fix a t in $0 < t \leqslant T$. For some fixed function $f(y)$ on R^d one considers the function $u(s, x)$ defined by

$$u(s, x) = \int f(y) p(s, x, t, dy) \quad \text{for } 0 \leqslant s \leqslant t. \tag{1}$$

Assuming that the function $u(s, x)$ is smooth, one shows that it satisfies the differential equation