finitely colored, then there is an infinite subset $S \subseteq N$ such that the set of all finite sums of distinct elements of S is monochromatic). Szemerédi's theorem (mentioned earlier) is shown to be a consequence of a theorem in ergodic theory by Furstenberg. One of the most interesting results in this chapter is the one by Harrington and Paris that the Ramsey type property (PH) on the integers is unprovable in Peano arithmetic. Property (PH) is the statement: For any positive integers n, k, and r, there is an m such that in any r-coloring of $[n, m]^k$ there is a monochromatic large set (a set $S \subseteq N$ is large if $|S| > \min(S)$). The authors complete the book with some selected results from the Ramsey theory of infinite sets.

One of the most miserable tasks of any author is to proofread the text. In this case, the task was taken seriously. The errors are minimal and most will be transparent to the reader.

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Spectral theory of ordinary differential operators, by Erich Müller-Pfeiffer, Ellis Horwood Series Mathematics and its Applications, Wiley, New York, 1981, 246 pp., $56.95.

With the study of Hilbert space having evolved into a challenging end in itself, it has become quite possible to overlook the connection between this area of mathematics and its underlying problems in differential equations. Such connections date back at least to the 1830's when Sturm and Liouville sought to generalize the idea of Fourier (sine) series to more general expansions in terms of the eigenfunctions associated with Sturm-Liouville equations of the form $-(pu')' + qu = \lambda\rho u$. While concepts such as "orthogonality" and "Fourier coefficient" generalize readily, questions regarding the convergence of these generalized Fourier series proved to be more difficult. Liouville's claims of rigor notwithstanding, it seems that questions of convergence were not properly formulated, much less rigorously resolved, until after 1900 [**2**].

The Hilbert space formulation of convergence provided more than rigor. It provided a context in which this question (and its trigonometric series specialization) has a simple and elegant solution, replete with necessary and sufficient conditions for such convergence. It also made the study of selfadjoint realizations of the formal operator $-\frac{d}{dx}p\frac{d}{dx} + q$ in the Hilbert space $L^2_\rho(I)$ a subject of profound importance.

E. Müller-Pfeiffer's *Spectral theory of ordinary differential operators* continues this study of selfajdoint realizations of Sturm-Liouville operators in terms of their natural generalization

$$a[\cdot] = \sum_{k=0}^{n} (-1)^k \frac{d^k}{dx^k} a_k(x) \frac{d^k}{dx^k}; \qquad a_n(x) > 0.$$

The coefficients are assumed to belong to an appropriate Sobolev class on an interval I of the form $x_0 < x < \infty$, where $x_0 \geqslant -\infty$. It is in this singular case, characterized by an infinite interval I, that the most interesting questions of spectral theory arise.

This book is not an introductory text on the subject. While it does indeed develop "advanced material from first principles", its Chapter 1 formulation of the underlying Hilbert space theory constitutes a mere 11 page presentation dealing with material at the level of Achiezer and Glazman's 1961 text **[1]**. With this brief but useful summary at hand, the author is able to turn quickly to the three topics of primary interest: (i) the location of the essential spectrum, (ii) criteria for a purely discrete spectrum, and (iii) criteria for the nonexistence of eigenvalues.

The first of these is dealt with in Chapter 2 on the basis of an explicit determination of the essential spectrum in the case of constant coefficients (i.e. $\sigma_e = [\Lambda, \infty)$ where $\Lambda = \inf_{0<\xi<\infty} \Sigma_{k=0}^{n} a_k \xi^{2k}$) followed by the study of perturbations which leave the essential spectrum invariant. It is in the application of these invariance criteria that one begins to see the craft of the contemporary researcher in this area—i.e. the detailed process of mollification and estimation which is required to apply the abstract theory. The chapter then considers the existence of eigenvalues below the essential spectrum and closes with a discussion of Euler (equation) operators in a similar vein.

Since discreteness of the spectrum is equivalent to $\sigma_e = \varnothing$ (or $\Lambda = \infty$), Chapter 3 continues the rather detailed process of estimation which is required to characterize operators of this type. Of interest here is the variety of criteria which are both necessary and sufficient for discreteness. The chapter closes with a brief consideration of periodic coefficients and of the case $n = 1$.

In Chapter 4 Professor Müller-Pfeiffer turns to operators with a purely continuous spectrum, and there arises for the first time the question of boundary conditions associated with various selfadjoint realizations. Here the problem is to show the absence of L^2 solutions in a class of related differential equations, and the author introduces a technique of scale transformations to facilitate the elaborate estimates required to achieve the very general results obtained.

Given the technical complexity inherent in Chapters 2–4, it is something of a relief to see the discussion specialized to the familiar Sturm-Liouville case ($n = 1$) in Chapter 5. In particular, this setting allows for very general results on the absence of eigenvalues.

A final chapter on oscillation theory is related to the main body of the book by the fact that $a[u] = \lambda u$ is oscillatory at $x = \infty$ if and only if the spectrum of any selfadjoint extension of $a[\cdot]$ on $C_0^\infty(1, \infty)$ is not finite "to the left of λ". This chapter presents a variety of generalizations of oscillation criteria of both "Kneser" and "integral" type. In particular, the Wintner-Leighton oscillation criterion is given an elegant generalization to equations of the form $(-1)^n(a_n(x)y^{(n)})^{(n)} + a_0(x)y = 0$. However now there is no simple proof in terms of Riccati transformations, but rather more of the detailed mollification and estimation which characterizes much of the book's text.

Spectral theory of ordinary differential operators presents a very current view of an important area of mathematics from the vantage point of one of the experts in the field. It is not encyclopedic, in the sense of Dunford and Schwartz [3], but instead presents a rather personal view. When it comes to a choice between developing the theoretical background vs. displaying the details of what is required to apply the theory, Professor Pfeiffer will tend to provide references to the theory but present the detailed calculations—ones which frequently represent his own signficiant contributions to the field.

The background and motivation for such hard work may have to be found in the texts and references to which the author frequently refers us. But since it very knowledgably brings the reader into contact with the techniques which are required to increase our understanding of such spectral theory, Professor Müller-Pfeiffer's book represents a valuable contribution to further research in this area.

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Combinatorial algebraic and topological representations of groups, semigroups and categories, by Ales Pultr and Věra Trnková, North-Holland Mathematical Library, Amsterdam, 1980, vi + 372 pp., $48.75.

The present subject is, of course, not much like group representation theory. But that can be said of most new subjects. Here there are two fundamental differences. First, one does not find distinguished representations, nor does one classify nice representations: the problem is existence. Think of Beltrami's Euclidean model of hyperbolic geometry. It proves (a) relative consistency, and (b) adequacy of the Euclidean theory of rather simple curves for formulating the Lobačevskian theory of straight lines. In the opposite direction, second, these are sharper representations: a group is represented as the group of *all* automorphisms of some structure; a category is represented by a number of structures of the same type and all their structure-preserving maps.

One might classify types of structure by their capacity for representing each other. If the familiar terms "hard", "soft", were to be given precise meanings, we might say "classify types of structure by their softness". What has actually been developed in about fifteen years is a theory of universally soft structures which can represent anything.

In the triad "groups, semigroups, and categories", the categories dominate. Certainly the facts that a group G is isomorphic with (1) a group of