$S(P^+)$. The next 160 pages treat well-known categories and how $S(P^+)$ can be embedded in them. The final chapter, 25 pages, presents strong universality theory. A most welcome 50-page appendix treats Cook continua, which are essential for showing that there is any universality in topology. (Cook's papers [**1, 2**] are much shorter—too much.) Finally, there is a brief survey of what happens if there exists a proper class of Ulam numbers.

The book jacket claims that no previous knowledge of category theory is assumed, and not much of other things. These claims are much better justified than such claims usually are.

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A history of the calculus of variations from the 17*th through the* 19*th century*, by Herman H. Goldstine, Studies in the History of Mathematics and Physical Sciences, vol. 5, Springer-Verlag, New York-Heidelberg-Berlin, 1980, xi + 410 pp., $48.00.

During my term at Cornell University my then colleague Mark Kac was fond of a quip which he had heard from Steinhaus who had heard it from Lichtenstein who was quoting Boltzmann: "elegance should be left to shoemakers and tailors". The classical single integral problems of the calculus of variations, the subject of this book, made a difficult and dirty field that even Bourbaki cannot make elegant. Possibly this is why the recent two volume history of mathematics in the 18th and 19th century, [**2**], which runs to about eight hundred pages, devotes three to the calculus of variations and seventy-eight to the theory of algebraic numbers. For Bourbaki the *raison d'être* for the older field seems to be the emergence of the concept of *functional* in the work of Volterra and Hadamard, and we all know what an elegant branch of mathematics emerged from that idea.

If this study had been written in the early forties—and it could have—it would have been a fitting memorial for a branch of mathematics which was born in the brachistochrone problem of the late 17th century, which reached its peak in the work of great men from Euler to Hilbert, and was finally entombed in Chicago in the thirties.

But, after Chicago, a miracle occurred. There was a second coming when, motivated by the needs of engineers and economists, the subject came to life again in the fifties in the field of *optimal control*. This development, which came too late for discussion in the present book, is beautifully treated in the exposition of Berkovitz, [**1**], who shows the relation between the old and the new.

The book under review is obviously a devoted labor of love. The author has not simply relied on previous accounts of the subject, but has gone back to the original works of the main contributors from Fermat through Hilbert and given a clear and meticulous account of their contributions. In this he has followed the precedent of two older and distinguished histories, those of Wodehouse, [**4**], and Todhunter, [**3**]. It is regrettable that (p. vii) he dismisses them as "hopelessly archaic". In saying this he is a mathematician, not a historian. That these books were written in 1810 and 1861 respectively only means that they could not give accounts of still undiscovered theory, but to the historian they are invaluable because they show what mattered at the time they appeared. St. Augustine's writing did not become archaic when some centuries later St. Thomas Aquinas wrote a "fresh" Christian theology.

Unfortunately the writing of the history of modern mathematics has not yet reached the maturity of the history of natural science. In the latter the development of basic ideas are pursued, so that a history book is not mainly a survey of the literature, i.e. an account of the important writings arranged chronologically. The reader of the present book will have difficulty tracing the history of basic ideas like transversality, conjugate points, or fields of extremals (or even their definitions) without first completing the tiresome task of checking through dozens of isolated references in the index. And while Hilbert's invariant integral is discussed in Chapter 7, i.e. in its proper place chronologically, its introduction earlier would have made it easier for a nonexpert to follow the work of Weierstrass; as it is, it receives only a passing mention in Chapter 5. The words "dynamics" or "dynamical systems" do not appear in the index, and the few pages devoted to Hamilton-Jacobi theory undervalue what may well be the major application of the calculus of variations in the nineteenth century.

Much patience and devotion has gone into the writing, but I cannot tell for whom this book is intended.

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