

BULLETIN (New Series) OF THE  
AMERICAN MATHEMATICAL SOCIETY  
Volume 17, Number 1, July 1987  
©1987 American Mathematical Society  
0273-0979/87 \$1.00 + \$.25 per page

*Positive operators*, by C. D. Aliprantis and O. Burkinshaw, Pure and Applied Mathematics, vol. 119, Academic Press, New York, 1985, xvi + 367 pp., \$59.00. ISBN 0-12-050260-7

The theory of vector lattices or Riesz spaces is a natural outgrowth of Lebesgue integration theory. It traces its origins to the work of Kantorovic and Freudenthal in the thirties and before that to a paper of F. Riesz in 1928.

Over the years the subject has undergone some changes of emphasis. Although closely allied to general linear functional analysis, in particular topological linear spaces, Banach spaces, and operator theory, it has remained distinct in its viewpoint. The development of the subject has, however, mirrored trends in functional analysis.

In the early days of the subject the theory of Banach lattices received some prominence. During the fifties and sixties the trend in functional analysis was towards increasing abstraction, and the fashion was to study locally convex spaces rather than Banach spaces. In vector lattice theory the same trend towards abstraction tended to produce the study of pure vector lattices without topological structure and of locally convex vector lattices. In some senses the marriage of locally convex spaces and lattices was natural and promising, as there is some interplay between order structure and topology. Indeed numerous researchers were attracted to the subject, and a number of books and monographs in the late sixties and early seventies resulted. Ultimately, however, there seemed to be little of real consequence resulting from this theory, and as fashions changed so vector lattice theory turned back to more concrete considerations.

Thus, in the last couple of decades, researchers have returned to questions concerning Banach lattices and positive operators between them. Probably the greatest influence in this direction was the work of Schaefer and his school, and, in particular, Schaefer's monograph [1] published in 1974.

In their current book, Aliprantis and Burkinshaw, who have been amongst the forefront of research in this area in recent years, are not seeking to compete with or replace Schaefer's book or the more recent book of Zaanen [2]. They aim instead to complement these books and concentrate on new developments in the eighties. Thus, they avoid some of the topics that one would normally expect in such a work. One finds no reference to spectral properties of positive operators (Perron-Frobenius type results) or to ergodic theorems. Unfortunately the omitted topics do include some of the best motivating examples for the general study of positive operators.

The authors spend the first hundred pages developing positive operators without any topological assumptions. The material in the first two chapters is more or less well known and common to several texts although the treatments vary and some modern wrinkles have been introduced. Then, after a short chapter on Banach spaces, they study some modern research developments in the final two chapters (about half the book).

Essentially, it may be said that the authors consider two related types of problems which have been of interest in recent research. The first is to find "domination" theorems. Thus if  $S, T: X \rightarrow Y$  are positive operators and  $S \leq T$  then one seeks to determine properties of  $S$  from those of  $T$ . This line was initiated in an important paper of Dodds and Fremlin in 1979, who showed that if  $Y$  and the dual of  $X$  have order-continuous norms and if  $T$  is compact then  $S$  is also compact. Since this paper there have been a number of ramifications of this general theme. The underlying idea is to prove, under suitable hypotheses, that  $S$  can be approximated by operators in the ideal generated by  $T$ .

The second type of question revolves around factorization theorems. A very useful result of Davis, Figiel, Johnson, and Pełczyński in general Banach space theory asserts that a weakly compact operator between two Banach spaces can be factored through a reflexive space. The analogue for positive operators and Banach lattices is false, as was shown by a recent counterexample due to Talagrand. Unfortunately, Talagrand's example is not presented in the book (although it is mentioned), perhaps because it appeared too late for inclusion. Nevertheless, there are a number of factorization results available and the authors emphasize their use. A typical example is a result, due to Aliprantis and Burkinshaw, that a product of two positive weakly compact operators can be factored positively through a reflexive Banach lattice. Theorems of this type and their relatives can be used to establish domination-type results.

The cycle of ideas represented in the final two chapters closely follows the research interests of the authors over the last few years, and many of the results are due to them. It seems to the reviewer that these problems are now well understood, and most of the results are in the best possible form.

In general, this is a careful and well-written account of certain aspects of positive operators. It is clearly not and was not intended to be a complete treatment; the reader is given an introduction to some specific parts of the general theory. For a more complete understanding of all the current trends one should also consult the works of Schaefer and Zaanen.

#### REFERENCES

1. H. H. Schaefer, *Banach lattices and positive operators*, Springer-Verlag, Berlin and New York, 1974.
2. A. C. Zaanen, *Riesz spaces. II*, North-Holland, Amsterdam, 1983.

N. J. KALTON

BULLETIN (New Series) OF THE  
AMERICAN MATHEMATICAL SOCIETY  
Volume 17, Number 1, July 1987  
©1987 American Mathematical Society  
0273-0979/87 \$1.00 + \$.25 per page

*On the Cauchy problem*, by Sigeru Mizohata, Science Press, Beijing, and Academic Press, Orlando, 1985, 177 pp., \$36.00 (paper). ISBN 0-12-501660-3

The theory of partial differential equations has as its source the study of a few model problems, many of them arising in physical applications. Laplace's equation and the wave and heat equations are prototypical, and the traditional