16. G. Reeb, *Variétés feuilletées, feuilles voisines*, C. R. Acad. Sci. Paris **224** (1947), 1613–1614.

17. ———, *Sur certaines propriétés topologiques des variétés feuilletées*, Hermann, Paris, 1952.

18. ———, *Structures feuilletées*, Differential Topology, Foliations and Gelfand-Fuks Cohomology (P. Schweitzer, ed.), Lecture Notes in Math., vol. 652, Springer-Verlag, Berlin and New York, 1978, pp. 104–113.

19. P. Schweitzer, *Counterexamples to the Seifert conjecture and opening closed leaves of foliations*, Ann. of Math. (2) **100** (1974), 386–400.

20. D. Struik, *A source book in mathematics 1200–1800*, Princeton Univ. Press, Princeton, N. J., 1986.

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Symplectic techniques in physics, by Victor Guillemin and Shlomo Sternberg, Cambridge University Press, Cambridge, London, New York, New Rochelle, Melbourne, Sydney, 1986, xi + 468 pp., $49.50. ISBN 0-521-24866-3

This book, aimed at a mixed public of physicists and mathematicians, starts with a 150-page chapter, called the Introduction, on optics. The purpose of this introduction is to illustrate the importance of symplectic geometry for physics.

The primitive way in which the discussion starts with Gaussian optics may serve as an eye-opener for some. However, it may also make a somewhat artificial impression on those who already have seen the relation between Snell's law, Fermat's principle, and Hamilton's treatment of geometric optics in some early physics course.

The subsequent epic story of Fresnel's discovery of the wave nature of light, leading to oscillatory integrals, is always very impressive. Here I am curious whether Fresnel himself really used complex notation as suggested in the book. Also I missed the end of the story, explaining light, and in particular its polarization, as rapidly oscillating solutions of Maxwell's equations.

Instead the book takes a surprising and exciting turn to use Fresnel integrals in order to pass to the standard representation of the metaplectic group on $L^2(\mathbf{R}^n)$. This move into quantum mechanics is followed by, among other things, a discussion of the Groenwald–van Hove theorem, saying that there is no way of extending the metaplectic representation to include any non-quadratic polynomial. It is a great service to the public to treat this subject so completely in this book.

Maxwell's equations do appear at the end of Chapter I, but only to treat the motion of a charged particle in an electromagnetic field as being Hamiltonian with respect to a symplectic form on the cotangent bundle, which differs from the standard one by a "magnetic term".

Chapter I is concluded with the provoking question "Why symplectic geometry?" I would like to add to their answer that classical mechanics not only is a high-frequency approximation to quantum mechanics, but also can be viewed as an aspect of partial differential equations of wave type, which is not inaccurate but rather loses some of the information without distorting it, in the way of a homomorphic image of a mathematical structure. For instance, the solution operator of a wave equation *is* a Fourier integral operator, with propagation of singularities described exactly by the corresponding classical Hamiltonian system.

The third fundamental principle proposed by the authors, that of general covariance, touches upon the recent increased interest in the study of moduli spaces of geometric structures, such as Riemannian (or complex) structures modulo the group of diffeomorphisms. Momentum maps and symplectic structures appear in a natural way in many cases. For instance, the moduli space for Riemann surfaces has a Kähler structure, the Weil-Petersson metric, which has turned out to be very useful.

As a general remark, I think that it is safe to say that mathematicians are quite convinced now that symplectic structures are at least as useful as, say, the Riemannian ones.

The remainder of the book is very different in structure from Chapter I; it contains primarily mathematical theory, applied only from time to time to physical examples. It starts in Chapter II with the geometry of the momentum map, in a very nice and complete description. In accordance with the philosophy that the book review should rather express the reviewer's ideas than describe the content of the book, I would like to give the following comments on the section on "collective motion", which is central to Chapter II and to part of Chapter IV on completely integrable systems.

First: As motivation, a Hamiltonian is discussed which has a term which is only a function of the total angular momentum and inertia tensor, and a potential with a sharp minimum at certain specific distances. This then results in "a rigid body motion together with a superimposed rapid oscillation." Actually, Hamiltonians of this kind have been proposed long ago as an explanation of Hamiltonian systems with constraints. The required asymptotic expansions have been worked out by, among others, Rubin and Ungar [**1**], see also Takens [**2**].

Second, the example of the spherical pendulum is shown to fall in the framework of collective motion. This observation does not help me in understanding the spherical pendulum better. I rather conclude from it that collectively integrable systems (or systems which are integrable because of symmetry, as I would call them) apparently can have quite interesting structure. In fact it strikes me that in the vast literature on integrable systems one often stops after having established that certain systems are integrable, instead of proceeding by taking a closer look at how they actually behave. In particular one would expect an analysis of the singularities of the fibration. The most singular fibers are the equilibrium points and the periodic solutions—in classical mechanics these special solutions traditionally got a lot of attention. In passing I cannot help but remark that the picture of the image of the momentum map for the spherical pendulum has been copied wrongly: it must have a corner at the minimum.

Third, if a system is integrable because of a noncommutative symmetry group, the phase space is fibered into isotropic tori of dimension smaller than the number of degrees of freedom of the system. From the point of view of perturbations ("breaking the symmetry"), this should be regarded as a degenerate situation. For instance the KAM theorem does not apply and no nearby quasiperiodic solutions will persist. (Only the singular fibers of the fibrations have more chance to survive.) This quite obvious remark about integrable systems is usually not made in the literature.

Returning to the actual content of Chapter II, the treatment of the convexity theorem, and the application to the asymptotics of multiplicities of representations in spaces of holomorphic sections of line bundles, is very nice, and remarkably light-footed in comparison to the highly nontrivial nature of the results.

Chapter III, containing among others the principle of covariance mentioned in the Introduction, treats the motion of a classical particle in a Yang-Mills field. The latter in turn is a sort of "classical substratum" of a quantum field theory that is more complicated than the "classical" quantum theory, which at least deals with linear operators. So in some sense Chapter III is two steps away from the real thing—if one insists on the latter.

In the meantime the moduli space for instantons (Yang-Mills fields satisfying a minimality condition) have been proved (by Donaldson and others) to give completely new information about the differential topology of 3- and 4-dimensional manifolds, a surprising new development in pure mathematics which was caused by an invention of theoretical physicists. As mentioned before, momentum maps appear in this context too.

Chapter III also contains a local normal form for Hamiltonian actions. This is one of the main new mathematical results in the book, found independently also by Marle. One would expect it to be useful also in the proof of the convexity theorems.

Chapter IV deals with Lie algebra techniques leading to complete integrability, such as the Kostant-Symes lemma. Systems of Calogero type are obtained as quotients of much simpler looking mechanical systems. The chapter ends with a pleasant review of some of the well-known infinite-dimensional integrable systems, such as KdV.

The last Chapter, V, is on filtered Lie algebras, which are deformed into their graded counterparts. The aim is to describe how the coadjoint orbits deform in the process, usually into families of lower-dimensional ones. For example, the wavelength is an invariant of light for the Galilean group, but not for the Poincaré group, due to the Doppler effect. Another spectacular application is Einstein's famous equation $E = mc^2$.

Chapter V also contains some basic standard facts about Lie algebras that are used throughout the book—it may be helpful to mention this here once again, because in the book cross-references are scarce.

Trying to write a book myself, I have great admiration for the fluent style of the authors. They have a lot of interesting ideas, and present them in a very eloquent way. A drawback is that often details are being glossed over. Also, the reader who wants to get full understanding could at several points have been helped with more specific references than those given in the text. Finally it is a pity that the reprint apparently has not been used to make corrections, because there are many minor, but sometimes irritating, misprints.

REFERENCES

1. H. Rubin and P. Ungar, *Motion under a strong constraining force*, Comm. Pure Appl. Math. **10** (1957), 65–87.

2. F. Takens, *Motion under the influence of a strong constraining force*, Global Theory of Dynamical Systems (Z. Nitecki and C. Robinson, eds.), Lecture Notes in Math., vol. 819, Springer-Verlag, Berlin and New York, 1980, pp. 425–445.

J. J. DUISTERMAAT

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Semigroups of linear operators and applications, by Jerome A. Goldstein. Oxford Mathematical Monographs, Oxford University Press, Clarendon Press, New York and Oxford, 1985, $x + 245$ pp., $42.50. ISBN 0-19-503540-2

This year is the centenary of the founding of the analytic theory of one-parameter semigroups: In 1887 Giuseppe Peano [**Pe**] wrote the system of ordinary differential equations

$$\begin{aligned} du_1/dt &= a_{11}u_1 + \cdots + a_{1n}u_n + f_1(t) \\ &\vdots \\ du_n/dt &= a_{n1}u_1 + \cdots + a_{nn}u_n + f_n(t) \end{aligned}$$

in matrix form as $du/dt = Au + f$ and found the explicit formula

$$u(t) = e^{tA}u(0) + \int_0^t ds\, e^{(t-s)A} f(s)$$

for the solution, where $e^{tA} = \sum_{k=0}^{\infty} t^k A^k/k!$. The mapping $t \geq 0 \to T(t) = e^{tA}$ is called the semigroup generated by A. More generally a C_0-semigroup on a Banach space $\mathbf{X}$ is a strongly continuous mapping from $\mathbf{R}_+$ into the bounded operators on $\mathbf{X}$ with the properties $T(t+s) = T(t)T(s)$ and $T(0) = 1$. The generator of T is the operator A defined by $Af = \lim_{t\to\infty}(T_t f - f)/t$ where f is in the domain $D(A)$ of A if and only if the limit exists. These concepts were introduced by Hille in the thirties, and he studied the semigroup by means of the resolvent $(\lambda - A)^{-1} = \int_0^\infty dt\, e^{-\lambda t}T(t)$. A fundamental theorem, proved by Hille and Yosida for contraction semigroups, and Feller, Miyadera, and Phillips for general semigroups around 1950, states that A is the generator of a C_0-semigroup T if and only if A is a closed, densely defined operator, and there exist real constants M, ω such that the resolvent $(\lambda - A)^{-1}$ exists for $\lambda > \omega$ and

$$\|(\lambda - \omega)^n (\lambda - A)^{-n}\| \leq M$$

whenever $\lambda > \omega$ and $n = 1, 2, 3, \ldots$. In this case $\|T(t)\| \leq Me^{\omega t}$, and

$$T(t)f = \lim_{n\to\infty}\left(1 - \frac{t}{n}A\right)^{-n} f \quad \text{for all } f \in \mathbf{X}.$$