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Classical sequences in Banach spaces, by Sylvie Guerre-Delabrière. Marcel Dekker, Inc., New York, 1992, xiv + 207 pp., $99.75. ISBN 0-8247-8723-4

For over sixty years a central question in modern analysis has been: Which Banach spaces contain *almost isometric copies* of one of the classical sequence spaces c_0 or l_p, for some $1 \leq p < \infty$? (A Banach space X contains *almost* *isometric copies* of a Banach space Y if for every $\varepsilon > 0$ there is an (into) isomorphism $T_\varepsilon : Y \to X$ with $\|T_\varepsilon\| \|T_\varepsilon^{-1}\| \leq 1 + \varepsilon$.) This problem originally surfaced in a different form. It is a simple observation (which even appears in Banach's book [2]) that $L_p[0, 1]$, $1 \leq p < \infty$, contains subspaces isometric to l_p. This observation evolved into a question well embedded into the "folklore" by the 1950s: Does every infinite-dimensional Banach space contain a subspace isomorphic to c_0 or l_p, for some $1 \leq p < \infty$? As absurd as this question seems today, it fit well into the naive world of Banach space theory of the 1950s. This was a time of hopeful optimism when one could imagine that every separable Banach space has a Schauder basis (*the basis problem*) or at least the approximation property (*the approximation problem*); that every infinite-dimensional Banach space was isomorphic to its subspaces of codimension 1 (*the hyperplane problem*); that every Banach space would contain an unconditional basic sequence (preferably the unit vector basis of c_0 or l_p) (*the unconditional basic sequence problem*); and surely, every Banach space X would have a subspace Y with a nontrivial projection (i.e., a bounded linear projection $P: Y \to Y$ with $\operatorname{rank} P = \operatorname{rank}(I - P) = \infty$).

In 1950 James [17] made a serious first step in this theory by proving that every nonreflexive Banach space with an unconditional basis must contain a copy of c_0 or l_1. Bessaga and Pelczynski [3] followed by showing that whenever c_0 embeds into a dual space X^*, l_1 embeds complementably into X, and hence l_∞ embeds into X^*. So there are no separable dual spaces containing c_0. Pelczynski [30] then showed that every infinite-dimensional subspace of c_0 (resp. l_p, $1 \leq p < \infty$) contains almost isometric copies of c_0 (resp. l_p). So far the optimism was well supported. But there was also a warning sign that all might not be in order. The awesome power of Grothendieck [15] had been brought to bear in an attempt to give a positive solution to the approximation problem—but to no avail. The year 1961 brought one of the most profound results in Banach space theory, showing that all was well—at least locally.

Dvoretzky's Theorem [8]. *Every infinite-dimensional Banach space contains almost isometric copies of* l_2^n, *for every* $n = 1, 2 \ldots$.

This was the first theorem in the "local theory" of Banach spaces which eventually split the field into two (highly competitive) parts, "local theory" and "infinite-dimensional theory". In 1964 James [18] made another important contribution: If X is isomorphic to l_1 (resp. c_0), then X contains almost isometric copies of l_1 (resp. c_0). This gave rise to the celebrated *distortion problem*: If a Banach space X has a subspace isomorphic to l_p, $1 < p < \infty$, must X contain almost isometric copies of l_p?

By the end of 1971 optimism had reached its peak. Lindenstrauss and Tzafriri [23] had given an affirmative answer to the *complemented subspaces problem*: If every subspace of a Banach space X is complemented, then X is isomorphic to a Hilbert space. The same authors [24] used the Banach fixed-point theorem to show that the Orlicz sequence spaces contain almost isometric copies of c_0 or l_p. This was the first class of spaces where the problem did not have an obvious answer. Although there was a small surprise here (there are Orlicz spaces which contain no complemented copies of l_p), the success of this approach convinced many people that the general problem had an affirmative answer.

In 1973 infinite-dimensional Banach space theory entered "the era of counterexamples". First, James [19] constructed *James's tree* (JT) as the first separable Banach space not containing l_1 with a nonseparable dual space. The big news, however, was that Enflo [9] had constructed a Banach space failing the approximation property. Grothendieck's research into the approximation problem [15] had established a connection with real analysis; namely, a counterexample to the approximation problem yields a counterexample to a question of Mazur from the "Scottish book" [26, problem #135]

> Given a continuous function $f(x, y)$ for $0 \leq x,\ y \leq 1$ and the number $\varepsilon > 0$, do there exist numbers $a_1, \dots, a_n$, $b_1, \dots, b_n$, and $c_1, \dots, c_n$ with the property that
> $$|f(x, y) - \sum_{k=1}^{n} c_k f(a_k, y) f(x, b_k)| \leq \varepsilon$$
> in the interval $0 \leq x,\ y \leq 1$?

While Enflo was traveling to Warsaw to receive his "live goose" on Polish television (the prize offered in the Scottish book), Davie, Figiel, and Szankowski (see [25]) were exploring the limits of these ideas. Thanks to their efforts we know that every l_p, $p \neq 2$, contains subspaces failing the approximation property. It is interesting that for $2 < p < \infty$ these were the first known examples of subspaces of l_p which are not isomorphic to l_p.

The dust had not even settled before Tsirelson [36] produced a surprisingly simple construction of a Banach space with an unconditional basis which contains no subspace isomorphic to c_0 or l_p, $1 \leq p < \infty$. The study of "Tsirelson-like" spaces quickly grew into an industry (see [7] for a complete treatment), and the problem it destroyed was rephrased as: Does every Banach space contain c_0, l_1, or a reflexive subspace? But 1974 also produced another profound result of the modern era—

Rosenthal's l_1-theorem [31]. *Every bounded sequence in a Banach space has either a weakly Cauchy subsequence or a subsequence equivalent to the unit vector basis of* l_1.

That is, given $c^{-1} \leq \|x_n\| \leq c$ in a Banach space X, either

(1) there is a subsequence (x_{n_i}) so that $\lim_{i \to \infty} f(x_{n_i})$ exists for every $f \in X^*$ or

(2) there is a subsequence (x_{n_i}) and a constant $K \geq 1$ so that $K^{-1} \sum_{i=1}^{\infty} |a_i| \leq \| \sum_{i=1}^{\infty} a_i x_{n_i} \| \leq K \sum_{i=1}^{\infty} |a_i|$ for all choices of scalars (a_i).

The introduction of spreading models by Brunel and Sucheston [4] gave new hope for a positive answer to the remaining classical problems. If (x_n) is a bounded sequence in a Banach space X, there is a subsequence (for simplicity denote it (x_n) again) satisfying

$$\lim_{\substack{n_1 < \cdots < n_k \\ n_1 \to \infty}} \left\| \sum_{i=1}^{k} a_i x_{n_i} \right\| \quad \text{exists}$$

for every sequence of scalars $(a_i)_{i=1}^{k}$. The subsequence we obtain is called a *spreading sequence*, and the infinite-dimensional Banach space it naturally defines is called a *spreading model* of X. By choosing (x_n) weakly convergent to

0, the spreading model has an unconditional basis and is finitely representable in the original space. Although spreading models are now a standard tool in the field, they did not resolve these classical problems. How close we can get to having c_0 or l_p inside a Banach space was established by a powerful result of Krivine [21]: If (x_n) is a basis for a Banach space X (or for c_0), there is a p so that the unit vector basis of l_p is *block finitely representable* on (x_n). In fact, for every natural number m and every $\varepsilon > 0$ there is a block basic sequence (y_n) in X (i.e., $y_n = \sum_{i=p_n+1}^{p_{n+1}} a_i x_i$, $p_0 < p_1 < \cdots$ natural numbers) so that

$$(1-\varepsilon)\left(\sum_{i=1}^{m} |b_i|^p\right)^{1/p} \le \left\|\sum_{i=1}^{m} b_i y_{n_i}\right\| \le (1+\varepsilon)\left(\sum_{i=1}^{m} |b_i|^p\right)^{1/p}$$

for every $n_1 < n_2 < \cdots$. For a complete treatment of spreading models, Krivine's theorem, and related "asymptotic properties" the best reference is [28].

For the next fourteen years the remaining classical problems kept their reputation for intractability intact. But in 1990 Schlumprecht produced a significant new example in Banach space theory [34, 35]. *Schlumprecht's space* S was much more than just another generalization of Tsirelson's space. In the skilled hands of Gowers and Maurey [14] it became the catalyst for the construction of Banach spaces containing no unconditional basic sequences. Johnson (see [14]) observed that a slight variation of these techniques produced *hereditarily indecomposable* Banach spaces (i.e., Banach spaces X for which no subspace has a nontrivial projection). The aura of invincibility for these classical problems was broken, and they were standing unprotected before an army of (now well-armed) giants. Although the outcome was predictable, the swiftness of the victory was not. Almost immediately Gowers [11] produced a Banach space with an unconditional basis not isomorphic to its hyperplanes. He followed this [12] with a Banach space which does not contain c_0, l_1, or a reflexive subspace. But he was not done yet. There was still the *Schroeder-Bernstein problem*: If two Banach spaces X and Y are isomorphic to complemented subspaces of each other, is X isomorphic to Y? Proving more, Gowers [13] constructed a Banach space X so that X is isomorphic to its "cube" but not its "square", i.e., $X \not\approx X \oplus X = Y$, but $X \approx X \oplus X \oplus X$. Not to be outdone, Schlumprecht and Odell [29] brought the classical "Mazur map" back to life to show that l_p, for $1 < p < \infty$, has arbitrarily large distortions. So there are Banach spaces isomorphic to l_p, $1 < p < \infty$, which do not contain almost isometric copies of l_p.

Among the few classical problems remaining open in this circle of ideas are:

The Homogeneous Banach Space Problem. If X is isomorphic to all of its infinite-dimensional subspaces (i.e., X is *homogeneous*), then must X be isomorphic to l_2?

The Unconditional Basis Problem. If every subspace of X has an unconditional basis, then must X be isomorphic to l_2?

Complemented Subspaces of Spaces with Unconditional Bases. Does every complemented subspace of a Banach space with an unconditional basis have an unconditional basis?

Compact Perturbations of the Identity. Is there a Banach space on which every bounded operator is a compact perturbation of the identity?

The L_1-Problem. Is every complemented subspace of $L_1[0, 1]$ isomorphic to l_1 or $L_1[0, 1]$?

The $C[0, 1]$ Problem. Is every complemented subspace of $C[0, 1]$ isomorphic to $C(K)$ for some compact metric space K?

Separable Quotients. Does every Banach space have a separable quotient space?

This new round of counterexamples led Vitali Milman to remark, "Is this the rebirth of infinite-dimensional theory or the final nail in its coffin?" This reviewer would view these as the end of the "naive" period and the beginning of the era of virtual reality. We can now go forward with a renewed optimism, steeped in the knowledge that Banach space theory is a much deeper subject than could have been envisioned at any time since its creation. And there are still basic structural results which hold. One example is a recent result of Rosenthal [33] which classifies those Banach spaces containing a copy of c_0 as those spaces containing a nonreflexive subspace Y so that every subspace of Y has a weakly sequentially complete dual space.

The book under review addresses the question, Which Banach spaces contain almost isometric copies of c_0 or l_p? Admittedly, we have left the impression that this question has no satisfactory answers. It is time to rectify this. In a deep paper in 1980 [1] Aldous showed that every subspace of $L_1[0, 1]$ contains almost isometric copies of l_q, for some q. Krivine and Maurey [22] then isolated the main ingredient in Aldous's proof to define the class of stable Banach spaces. A separable Banach space X is *stable* if, for all bounded sequences (x_n) and (y_n) in X and all nontrivial ultrafilters $\mathscr{U}$, $\mathscr{V}$ on the natural numbers, we have

$$\lim_{n,\mathscr{U}} \lim_{m,\mathscr{V}} \|x_n + y_m\| = \lim_{n,\mathscr{V}} \lim_{n,\mathscr{U}} \|x_n + y_m\|.$$

Many naturally arising spaces are stable, including L_p-spaces for $1 \le p < \infty$ [22] (c_0 and $L_\infty[0, 1]$ are not stable). Chapter III of the book is devoted to a detailed proof of the Krivine-Maurey result that stable Banach spaces contain almost isometric copies of l_p, for some $1 \le p < \infty$. Clearly, subspaces of stable spaces are stable, so this contains the Aldous result. There is also another result of Krivine and Maurey in Chapter III: $L_p(X)$ is stable if X is stable. Therefore, every subspace of $L_{p_1}(L_{p_2}(L_{p_3}(\cdots)))$ contains almost isometric copies of l_q, for some q.

Chapter II contains the necessary background on spreading models and ultrapowers as well as Krivine's theorem on the block finite representability of c_0 or some l_p on every basis.

Chapter IV addresses the question of *which* l_q's embed almost isometrically into a subspace of L_p. There is the Kadec-Pelczynski result [20] that every infinite dimensional subspace of $L_p[0, 1]$ (for $2 < p < \infty$) is either isomorphic to l_2 or contains almost isometric copies of l_p. For $1 < p < 2$ the situation is complicated by the fact that l_q is isometric to a subspace of $L_p[0, 1]$, for all $q \in [p, 2]$ (and no other l_r embeds into $L_p[0, 1]$). But there we have the *Guerre-Levy theorem* [16] which identifies precisely our copy of l_q inside a subspace of $L_p[0, 1]$; namely, if X is an infinite-dimensional subspace of L_p,

$1 < p < \infty$, then $l_{p(X)}$ is almost isometric to a subspace of X where

$$p(X) = \sup\{p \in [1, 2] | X \text{ is of Rademacher type } p\},$$
$$q(X) = \inf\{q \in [2, \infty) | X \text{ is of Rademacher cotype } q\}.$$

This is the infinite-dimensional version for $L_p[0, 1]$ of the *Maurey-Pisier Theorem* [27]: Every Banach space X contains almost isometric copies of $l^n_{p(X)}$ and $l^n_{q(X)}$, for every $n = 1, 2, \ldots$.

The case $p = 1$ is covered by the Aldous result. But prior to this, Kadec and Pelczynski [20] showed that an infinite-dimensional subspace of $L_1[0, 1]$ is either reflexive or contains almost isometric copies of l_1. The elegant result of Rosenthal [32] is not proved in the book. It states that a subspace of $L_1[0, 1]$, which does not contain l_1, must embed into L_p, for some $1 < p \leq 2$.

Chapter I of the book contains proofs of several, now classical, results. This chapter includes proofs of

(1) Rosenthal's l_1-theorem,
(2) the Bessaga-Pelczynski characterization of Banach spaces containing c_0,
(3) James's proof of the nondistortability of c_0 and l_1,
(4) the existence of subspaces of l_p (for $2 < p < \infty$) without the approximation property, and
(5) the Figiel-Johnson construction of Tsirelson's space [10].

Many of the techniques developed in this book have heretofore been available only in research papers. The terse and concise presentation helps to keep the technical details from obscuring the ideas but may require some adjusting to. The book is intended for graduate students and is reasonably well self-contained. Although the depth of the material will hamper rapid progress, the rewards will include a glimpse at some of the most beautiful and significant results in the field and exposure to techniques quite different from those one usually encounters in modern analysis. Also, students will benefit from exposure to the important theory of ultraproducts.

Although the book contains a number of typographical errors, they are obvious enough so as not to be a serious impediment. (One of them, however, rendered Proposition I.1.13(iii) false as stated. It should read: $\overline{\text{span}}[x_n : n \in N]$ and $\overline{\text{span}}[y_n : n \in N]$ are *naturally* isomorphic. That is, the operator defined by $Tx_n = y_n$ is an isomorphism.)

Finally, the reviewer would have preferred to see more attention to detail in the referencing of the material. Here are just a couple of examples:

(1) Saying that the proof of Corollary I.7.4 comes from a book of Beauzamy and Lapresete disguises the fact that the proof *there* is due to Figiel and Johnson [10] (in fact, this "secondary referencing" technique is very heavily employed in the book).

(2) Despite the statement on p. 35 that Paley gave the value of the unconditional constant for the Haar system in his 1932 paper, the value was not discovered until fifty years later (Burkholder [5, 6]).

Just before this book review appeared, two recent papers combined to produce a positive answer to the *homogeneous Banach space problem*. First,Ryszard Komorowski and Nicole Tomczak-Jaegermann [39] showed that a homogeneous Banach space which contains an unconditional basic sequence is isomorphic to a Hilbert space. Next, W. T. Gowers [38] proved that a Banach space which contains no unconditional basic sequences must contain a hereditarily indecomposable subspace. Since it was known (see [37]) that homogeneous Banach spaces are decomposable, it follows that every homogeneous Banach space is isomorphic to a Hilbert space.

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Topics in Galois Theory, by J.-P. Serre. Research Notes in Mathematics, 1992, Jones and Bartlett Publishers, xvi+116 pp. ISBN 0-86720-210-6

Serre's book *is* a set of topics. It contains historical origins and applications of the inverse Galois problem. Its audience is the mathematician who knows the ubiquitous appearance of Galois groups in diverse problems of number theory. Such a mathematician has heard there has been recent progress on the inverse Galois problem. Serre has written a map through the part of this progress that keeps *classical landmarks* in sight. We will describe Serre's view of present achievements toward that goal and comment on the territory he ignored. We will denote Serre's book by [Se] throughout the review.

Galois theory is the supreme topic in an area once called the *Theory of Polynomials*. Versions of the inverse Galois problem have immediate application in algebraic number theory, arithmetic geometry, and coding theory. This includes applications driven by the theory of finite fields. Until recently, however, attacks on the problem were ad hoc. Even when general approaches arose in the late 1970s, acceptance took a long time. Then special approaches still held promise. Examples now show why earlier methods will not solve the complete problem.