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Operator-limit distributions in probability theory, by Zbigniew J. Jurek and J. David Mason. Probability and Math Statistics, Wiley, New York, 1993, xii + 292 pp., $79.95. ISBN 0-471-58595-5

The central limit theorem is one of the cornerstones of probability theory. The importance of the central limit theorem is that it allows the probabilistic behavior of the sum of a large number of independent random variables to be approximated by the probabilistic behavior of a single random variable which is often simpler (in some sense) than the summands themselves. This is especially true in the case of the familiar statistician's central limit theorem. This result states that if $\{X_j\}$ is a sequence of independent and identically distributed random variables with finite variance, then for large n the distribution of the sum $\frac{1}{\sqrt{n}}\sum_{j=1}^{n}(X_j - E[X_j])$ is approximately a normal distribution.

There are at least two useful directions for generalization of the statistician's central limit theorem. First, the assumption that the $\{X_j\}$ are identically distributed and have finite variance can be relaxed. Second, the random variables $\{X_j\}$ can be replaced by random vectors. Both of these directions are natural from a statistical point of view, since each observation may record several measurements on a single experimental unit.

Exploration of both of these directions leads to the following problems. Given a sequence $\{X_j\}$ of independent random vectors, find conditions under which there are linear operators $\{A_n\}$ and vectors $\{a_n\}$ so that

$$A_n \sum_{j=1}^{n} X_j + a_n$$

converges in distribution to some random vector Y. Also, characterize the random vectors Y which can arise in this way, identify the relationship between

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Y and $\{X_j\}$, and give a "formula" for computing A_n and a_n. (In all of this discussion it is required that the limit random vector Y not be supported in a proper hyperplane.) These are the broad issues addressed in the book under review.

In the generality of the problem stated in the previous paragraph, the limit random vectors Y which can arise are said to be operator self-decomposable (or in the univariate case, in class L). When the sequence $\{X_j\}$ is additionally assumed to be identically distributed, the limit random vectors Y are said to be operator stable. Operator stable random vectors are the multivariate analog of stable random variables taking values in $\mathbf{R}$, of which the normal and Cauchy types are the most familiar.

When all of the random vectors are real valued (the one-dimensional case), all of the questions raised above received a complete treatment in the book by Gnedenko and Kolmogorov [GK]. When the random vectors take values in $\mathbf{R}^d$ for some $d > 1$, or take values in an infinite dimensional Banach space, many of the central questions are still unanswered. Indeed, this is precisely the area of active development surveyed by this book. The main objectives of the authors are to collect the core part of the theoretical development for $\mathbf{R}^d$ valued random vectors and to highlight the differences in the theory in the finite- and infinite-dimensional cases.

The authors skillfully develop the prerequisite material on infinitely divisible probabilities, probability on Banach spaces, Lie theory, and semigroup theory, presenting the key results without proof. Following this prelude, most of the known results to date about the above-mentioned problems are presented, often with proofs that are much more understandable than those in the original papers. In particular, facts about the normalizing sequence $\{A_n\}$ are developed, the special form of the characteristic function of operator self-decomposable and operator stable random vectors is given, and operator self-decomposable random vectors are shown to have a representation as random integrals. For operator stable random vectors, more detailed structure of the characteristic function is presented, and the domain of normal attraction (the $\{X_j\}$'s that can arise when Y is given and A_n is of a special form) is characterized. The authors have maintained their focus on these problems, mentioning other related topics (such as absolute continuity with respect to Lebesgue measure, existence of moments and independent marginals) as the need arises but not wandering from their intended purpose. This yields a well-written and easily readable treatment leading from the modern beginnings of the subject some twenty-five years ago in the work of Urbanik [U] and Sharpe [S] to the forefront of current research in the field.

Readers of the book should have some knowledge of measure theoretic probability, but extensive experience with distributional limit theory is not required. The book is ideal for use in a graduate seminar or for researchers desiring to be quickly brought up to date on the above aspects of the multivariate central limit theorem.

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Model theory, by Wilfrid Hodges. Encyclopedia of Mathematics and its Applications, vol. 42, Cambridge University Press, Cambridge, 1993, xiii + 772 pp., $99.95. ISBN 0-521-30442-3

Mathematical logic began as the general study of mathematical reasoning. Several specializations have developed: recursion theory studies abstract computation; set theory studies the foundations of mathematics as formalized in Zermelo-Fraenkel set theory; proof theory studies systems of formal proof; model theory, says Hodges, is the study of the construction and classification of structures within specified classes of structures. Despite this somewhat jargon-laden definition, model theory is the part of logic that over the last half century has developed the deepest contacts with core mathematics. I would modify Hodges's definition to emphasize a central concern of model theory: the discussion of classes of classes of structures. In this review we consider several developments, most covered by Hodges, to clarify the distinction between studying a single structure, a class of structures (a theory), or a class of theories. We will relate these model-theoretic concepts to specific developments in the theory of fields.

A signature L is a collection of relation and function symbols. A structure for that signature (L-structure) is a set with an interpretation for each of those symbols. Consider a signature L with symbols: $<$, $+$, $\cdot$. The ordered field of real numbers becomes an L-structure when these symbols are given their natural interpretations. A language specifies certain L-sentences built up from the symbols in a signature L. A number of languages can be built up based on the same signature. The first-order language is the least set of formulas containing the atomic L-formulas (e.g., $x < y$) and closed under the Boolean operations and quantification over individuals. Formulas in which each variable is bound by a quantifier are called L-sentences. More expressive languages allow closure under infinite conjunctions ($L_{\kappa,\omega}$), quantifiers over subsets (second-order logic), etc. An L-structure M is a model of an L-sentence if the sentence is true in M. Thus the class of real closed fields is axiomatizable in the first-order language associated with L, while the reals are axiomatizable in the second-order language associated with L.