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Normal forms and bifurcations of planar vector fields, by S.-N. Chow, C. Li, and D. Wang. Cambridge University Press, Cambridge and New York, 1994, viii + 472 pp., $69.95. ISBN 0-521-37226-7

This book studies planar vector fields. The phase portraits of such fields contain a kind of concise, non–verbose information similar to e.g. city–maps. Vector fields show up as dynamical systems, which in general need not have a 2–dimensional phase space. However, there are many situations that allow reduction to this special case of dimension 2.

A familiar reduction principle, e.g., known from physics, uses symmetry. An example within reach of the techniques of the book is the following. Consider a periodically forced oscillator of the form

$$\ddot{x} = f(x, \dot{x}, t), \quad x \in \mathbb{R}, \ t \in \mathbb{R},$$

say with period 1 in the time t, of which we like to study resonances. A corresponding dynamical system is

$$\dot{x} = y, \ \dot{y} = f(x, y, t), \ \dot{t} = 1,$$

which has a 3–dimensional phase space $\{x, y, t\}$. One way to deal with this is to look at its Poincaré or stroboscopic map P, which is again 2–dimensional. By definition it follows the associated 3–dimensional flow from $((x, y), 0)$ to $(P(x, y), 1)$, so from the section $t = 0$ to $t = 1$. Thus a fixed point of P corresponds to a periodic solution of period 1, etc. Now a general 2–dimensional map can still be quite a complicated object, but fortunately the present problem is to study the Poincaré map in the neighborhood of such a fixed point, say p. If S denotes the semisimple part of the derivative $D_p P$,

then a normal form theorem formulated by Takens (1974) says that – up to an appropriate change of variables – an approximation

$$P \approx \Phi_X^1 \circ S$$

holds in a sense that can be stated precisely in terms of a Taylor–formula. Here X is a planar vector field, while Φ_X^1 denotes its flow over time 1. Moreover X is S-symmetric in the sense that $SX(x,y) \equiv X(S(x,y))$. The fact that the t–coordinate has disappeared in the approximation amounts to a rotational symmetry. It may be clear that this planar field X (and its phase portrait) has a lot to say about P.

In the case of $p : q$ resonance the matrix S simply is a rotation over angle $2\pi p/q$. For a proper study of the system one now wishes to include parameters (detuning, etc.) in the problem. The strongly resonant cases $q = 1, 2, 3$ and 4 are the most interesting, and they are all described in the book, as is the remaining case, $q \geq 5$. This description is completely self–contained and covers most of the (sometimes very recent) literature on this subject.

The above example motivates the local study of planar vector fields with discrete symmetry. There exist other normal form techniques that reduce certain 3- or 4-dimensional vector fields to planar fields with reflectional symmetries. This concerns equilibria of vector fields whose linear parts have purely imaginary eigenvalues. Now a circle viz. a 2-torus symmetry is factored out; see [1], [2]. Also these cases, with the appropriate parameters included, are described in the book.

The second chapter of the book deals with all the above normal forms in an extensive way. Normal forms, in this nonlinear context, are adjustments of the Taylor–series at an equilibrium (or fixed point), carrying out successive changes of variables. This procedure uses the linear part in the equilibrium and involves linear algebra on spaces of homogeneous polynomials. Variations of this approach are given in the Hamiltonian case and in cases with simple symmetries.

Preceding the normal forms, the book starts with another reduction principle, using the center–manifold. A rough idea of this is the following. Near a higher–dimensional equilibrium point, the strongly (i.e., exponentially) contracting and repelling dimensions, for all dynamical purposes, can be decoupled and forgotten. The corresponding eigenvalues are called hyperbolic, and what remains is the center–manifold. The existence and, e.g., non–uniqueness of this object are treated in detail.

After this an exposition is given of Arnol′d's theory of matrices depending on parameters; cf. [1]. This theory gives a systematic way of introducing parameters into a linear system $\dot{x} = Ax$, $x \in \mathbb{R}^n$, in order to describe the dynamics in all nearby linear systems, the parameters serving to deform or unfold the system. This theory has quite a geometric flavour, as can be seen from its key term "transversality". Again variations of the theory are presented in the Hamiltonian (infinitesimally symplectic) case and in cases with a simple symmetry.

The remaining half of the book deals with planar vector fields. Part of the above machinery can help to get into the plane, but once in the plane the machinery can be of further use to understand the nonlinear dynamics near a degenerate equilibrium. Therefore many neighboring systems have a different dynamic, and one introduces parameters into the linear part as before.

A complete treatment is given of the generic local classification problems in the cases where 1 or 2 parameters are involved. The theory speaks of "bifurcations of codimension 1 viz. 2". The corresponding families of vector fields are classified up to (topological) equivalence. This includes all cases mentioned earlier. The final part of the book gives an up–to–date picture of bifurcations with higher codimension. The whole program is particularly of interest in the light of Hilbert's 16–th problem.

A major difficulty is the finding of limit cycles, which by a Mel'nikov–like method usually leads to Abelian integrals and corresponding Picard Fuchs equations.

The philosophy of the book is inspired by the question what a researcher, new to the field, might want to know. He is assumed to have ample knowledge of analysis and differential equations, but given this, the text is largely self–contained. This means that proofs have to be as elementary and direct as possible, using e.g. no differential geometry nor (graded) Lie algebra nor a coordinate free approach to Hamiltonian systems. The authors have gone to great pains to achieve this, thus providing many proofs that were not available in the literature before. As an example of this, see their treatment of the Bogdanov–Takens bifurcation, i.e., the resonance case mentioned before with $q = 1$. Often this style of writing requires more technicalities than in the known treatments, but sometimes also the new proof is surprisingly short!

Here I add a remark. Both the normal form theory and the Arnol'd theory of parameter–dependent matrices allow generalisations in a unifying Lie–algebraic setting, which makes it possible to understand various special cases at once (this includes all cases presented in the book). Of course such a presentation is outside the present scope. Nevertheless I would have preferred the reader to be encouraged to try and master some of this sophistication, since this pays, even if for only technical purposes. The book does contain an extensive bibliography and each chapter ends with bibliographical notes, but perhaps in a later version remarks in this direction could be added.

In general I feel the book to some extent compares to Palis and de Melo [3], although there are a lot of differences too. For one thing, both texts are largely self–contained introductions to dynamical systems at a research level, dealing with genericity and structural stability. The contents also have a large complementary component, since [3] aims at the Kupka–Smale theorem. Finally, the style is quite different, since Palis and de Melo are more geometrically oriented. This shows, for example, from their use of the graph transform instead of the variation of constants formula, which frequently occurs in the book under review.

In summary and despite the above criticism, I cordially recommend the book to researchers new to this field.

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Group theory and physics, by S. Sternberg. Cambridge University Press, Cambridge, 1994, xiii + 429 pp., $69.95. ISBN 0-521-24870-1

Textbooks for an elementary course in calculus or differential equations aimed at science and engineering students almost invariably stress the physical motivation underlying the subject matter. Examples and exercises naturally focus on the older and more commonly encountered parts of physics such as classical mechanics and electromagnetic theory. When the students are drawn from a wider range of disciplines such as economics and biology, the motivating examples have to be adjusted accordingly—and may become somewhat less persuasive due to artificial or oversimplified formulations.

The urge to justify mathematics to students by pointing to "real world" applications has led to more frequent inclusion of such topics as coding theory in recent textbooks on linear algebra and abstract algebra. But older books on algebra, or those written for graduate students, generally suppress any hint of applicability outside mathematics itself. In particular, most accounts of abstract group theory or group representations written by mathematicians ignore the rich applications of these subjects to twentieth century physics and chemistry. (One exception is the recent text [JL] on finite group representations by James and Liebeck, whose concluding chapter applies character theory to molecular vibration.)

Symmetry is an ever-present consideration in the study of natural phenomena. Group theory provides a precise language with which to describe the possible symmetries of a physical system. In the case of crystallographic groups, the successful classification of relevant symmetry groups in the nineteenth century made possible many predictions of what may actually be observed in nature. But modern physics sometimes requires more subtle data coming not just from the obvious geometric actions of groups but also from the possible representations of the groups by linear operators (typically acting on spaces of functions).

This is especially apparent in the study of elementary particles since the 1960s, leading to what is now known as the Standard Model. As this review is being written, news media are reporting confirmation by two research teams of the existence of the elusive (and massive) "top" quark. But neither the mass media nor the books by physicists intended for very general audiences give any clear impression of just what kind of mathematics is involved in the quark theory. Sternberg's book goes a long way in this direction, for a mathematically oriented reader.