suggestions for further reading at the end of the book): mysterious citations in the text such as "see [WW80]" on page 301 lead nowhere. This is being repaired in the softcover reprinting now underway, according to the publisher.

Such faults aside, the book is the next best thing to having a long chat with the author about subjects which are obviously near to his heart. He has made a valuable contribution to the breaking down of artificial barriers between mathematics and physics.

REFERENCES

[C] J. F. Cornwell, *Group theory in physics*, Academic Press, Orlando, FL, 1984.

[JL] G. D. James and M. W. Liebeck, *Representations and characters of groups*, Cambridge Univ. Press, Cambridge, 1993.

[T] Wu-Ki Tung, *Group theory in physics*, World Scientific, Singapore, 1985.

J. E. HUMPHREYS
UNIVERSITY OF MASSACHUSETTS
E-mail address: jeh@math.umass.edu

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 32, Number 4, October 1995

Green functions for second order parabolic integro-differential problems, by M. G. Garroni and J. L. Menaldi. Longman Scientific & Technical, Harlow, Essex, England, 1992, 417 pp., $57.00. ISBN 3-582-02156-1

The topic of this book is the deep and highly involved treatment of parabolic equations of second order containing a nonlocal term of a special structure. This treatment is general in the sense that all types of boundary conditions are considered with equal weight, both in Hölder- and L_p-spaces.

The equation is $Lu - I(u) = f$, where

$$Lu = \frac{\partial u}{\partial t} - a_{ij}(x, t)\frac{\partial^2 u}{\partial x_i \partial x_j} - a_i(x, t)\frac{\partial u}{\partial x_i} - a_0(x, t)u$$

is a classical linear uniformly parabolic operator of second order with bounded $(\alpha, \frac{\alpha}{2})$-Hölder continuous coefficients. Chapter 1 collects all the standard material (Schauder estimates in $C_{\alpha,\alpha/2}$ and L_p, maximum principles, etc.) concerning L. The nonlocal operator $I(u)$, modelling jumps in the diffusion process, has (roughly) the structure

$$I(u) = \int_F [u(x+j(x, t, \xi), t) - u(x, t) - j(x, t, \xi)\cdot\nabla_x u(x, t)]m(x, t, \xi)\pi(d\xi)$$

with some σ-finite measure π on F, and $T_\theta(x) = x + \theta j(x, t, \xi)$ is a diffeomorphism for all $\theta \in [0, 1]$, t, ξ. The properties of I are discussed in Chapter 2; especially conditions are given such that $\|I(u)\|_\chi \le \varepsilon\|D^2 u\|_\chi + c_\varepsilon$ l.o.t. for $\chi = C_{\alpha,\alpha/2}$ or L_p. Due to the "minus" sign in front of $I(u)$ the maximum principle can also be saved; hence this would be enough for just solving the equation via fixpoint arguments.

But the aim of the authors is a different one, as indicated in the title of the book. Hence its middle part (Chapter 3–Chapter 6) gives a dense presentation, construction and discussion of the properties of the fundamental solution and of the Green functions for L under various boundary conditions. One can find the whole classical material, including simple and double-layer potentials, jump relations and Levi's Parametrix method. The other possibility for constructing the Green function, using the existence theory for the initial boundary value problem for $Lu = f$, is indicated in the third part of Chapter 6.

The last three chapters are devoted to the construction and discussion of the Green functions G for $L - I$. The key tool for the construction of G as a series of terms, each of which solves a Volterra equation, is the definition and study of a certain scale of Banach-spaces containing these kernel functions. This scale is defined by means of fifteen(!) (semi-)norms. Chapter 8 contains the construction of G, and properties and estimates of G are discussed in Chapter 9.

Due to the subject, the contents of the book are highly technical—a research note in the best sense. It has the further merit of containing the classical material for linear parabolic equations with variable coefficients for all types of boundary values in a form which allows a citation.

Michael Wiegner
Institut für Mathematik
Universität Bayreuth
E-mail address: michael.wiegner@uni-bayreuth.de

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 32, Number 4, October 1995

Elliptic differential equations (*theory and numerical treatment*), by W. Hackbusch (translated from the German by Regine Fadiman & Patrick D. F. Ion). Springer-Verlag, Berlin, Heidelberg, and New York, 1992, xiv+311 pp., $89.00. ISBN 3-540-54822-X

This book gives a rigorous account of the theory of elliptic partial differential equations and numerical approximations. There are some gaps in the sense of missing material that one would have thought would be in a book like this. However, the material that is covered is for the most part very well done. A lot of attention and care has been given to both proofs and examples, and the result is a highly readable volume that is suitable for advanced undergraduates and beginning graduate students.

The first three chapters deal with scalar second-order elliptic equations. The approach is classical with an emphasis on the mean value property, related integral representation, and maximum principles. The topology used is defined by the sup norm, and various properties of classical solutions are established. These include existence, uniqueness, and continuous dependence on data.