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*Continuum percolation*, by Ronald Meester and Rahul Roy, Cambridge Univ. Press, 1996, x + 238 pp., \$49.95, ISBN 0-521-47504-X

From the outset, mathematical percolation theory has been motivated by problems in continuous random media such as the following, which are mentioned in Broadbent and Hammersley's pioneering 1957 paper [1] on the subject:

1. At what density of pores (holes) in a large porous rock does it become possible for a fluid to flow through the rock?
2. If one tree in an orchard becomes diseased, and if disease may pass between a pair of trees with probability depending on the intertree distance, what is the chance that a faraway tree in the orchard will be affected?

Most mathematical research in the subject, described in Grimmett [2], for example, has been concerned with lattice models. However, questions in the continuum, as suggested by the above problems, have been of growing interest to mathematicians. Meester and Roy's book is the first to be devoted to these developments.

Much of the book is concerned with the so-called *Boolean model* for Problem 1 above, in which the pores are modeled by balls of random radii centered at the points of a spatial Poisson process. The components of the union of the balls are the "occupied clusters"; "vacant clusters" are also considered. By contrast with lattice models these tend to require distinct arguments.

The book also discusses the so-called *random connection model* for Problem 2 above, in which each pair of Poisson points is connected with probability depending on their separation, inducing a random graph whose components are "clusters". A further chapter is concerned with generalizing from the Poisson process to more general stationary point processes; here ergodic theory comes into play. A variety of other models are also briefly discussed in the final chapter. Nevertheless, the main concerns are the Boolean and random connection models on a Poisson process of rate $\lambda$.

One major issue in the book is that of phase transition. The percolation theorist is interested in large clusters and especially in the critical value of $\lambda$ above which there are *infinite* clusters. Also of interest is the critical value of $\lambda$ above which the *mean* cluster size is infinite. In the Boolean model there are several ways to describe cluster size; thus there are several ways to define the critical value of $\lambda$, and quite a bit of work has been required to show that they all give the same critical value of $\lambda$ when the balls are of bounded radius. If critical values are defined in terms of *vacant* clusters, their equality has still not been proved for more than two dimensions.

Another question studied in that of uniqueness; there is never more than one infinite cluster. This is established both for occupied and for vacant clusters, and for the random connection model. These questions of phase transition and uniqueness are analogues to those which have been considered on the lattice and have analogous answers, at least when the radii of the balls are bounded; however, the range of techniques required goes beyond mere discretization and application of known lattice techniques. The RSW lemma and BK inequality, for example, have proved

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particularly challenging in the continuum. The technicalities are dealt with in a clear manner which should benefit researchers in the field.

Some results in the continuum, for example, on the spatial distribution of finite clusters, have no obvious discrete analogue. At high Poisson density, most of the Poisson points lie in the infinite cluster, while the remaining finite clusters have a compressed structure in the large-$\lambda$ limit, which is described in some detail.

Boolean models in which more general shapes than balls are used have previously been discussed in the books on stochastic geometry by Hall [3], and Stoyan, Kendall, and Mecke [4]. Meester and Roy have chosen to restrict their attention to balls, except for a single section on "Poisson sticks" at the end, and this has simplified the presentation. Their exposition of more percolation-theoretic results, such as those discussed above, complements the material in [3] and [4] without overlapping it. Unlike the authors of [3] and [4], they do not discuss statistical issues such as the estimation of $\lambda$.

Meester and Roy's approach is to treat the subject as a worthwhile mathematical pursuit in itself and to present the theory in a rigorous, coherent, self-contained, and readable manner. In this they have succeeded admirably, and the book will be the standard reference for any researcher in continuum percolation. For this coherence, however, they may have paid a price in terms of wider appeal. There is little discussion of applications and connections to other subjects, which is a pity, because the subject material could interest and stimulate researchers from both the statistics and physics communities.

On the statistical side, the relation to stochastic geometry has already been discussed. In addition, continuum percolation is a valuable tool in the study of large-sample properties of structures used in multivariate analysis, such as single linkage clusters and minimal spanning trees.

From the statistical physics point of view, the subject might at first sight appear to be less compelling because of qualitative similarities to simpler lattice models; on the other hand, the development of a theory for rotationally invariant continuum models should be of interest in the context of such issues as conformal invariance, and some will find the greater physical realism of continuum models in some contexts sufficient motivation in itself for their study.

This book is a timely synthesis of the developments in an interesting field. The style and organization are excellent. The book should be very helpful for researchers in the area, and also has something to offer to those with interests in related areas such as stochastic geometry and lattice percolation.

## References

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