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*Cours d'analyse mathématique*, by Édouard Goursat, Professeur à la Faculté des Sciences de Paris, Tome I, Gauthier-Villars, Paris, 1902

The revision of the fundamental principles of the calculus, which was initiated by Cauchy and Abel and carried through by Weierstrass and his followers, led to the development of the $\epsilon$-proof (early introduced by Cauchy) and to the precise formulation of definitions and theorems. In Germany and Italy a tendency sprang up to place only such restrictions on definitions and theorems as are necessarily imposed by the nature of the case. Thus functions continuous throughout no interval whatever were admitted as the integrand of a definite integral simply because the form of the definition of the integral applied to a certain class of these functions, and the question was examined of how far the ordinary theorems of the integral calculus hold for such functions. Again, the theorem that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ was proved with fewer restrictions than the continuity of all the derivatives that enter. While this procedure is perfectly justifiable so far as it is a question of research in a special field, it is important not to lose sight of the fact that investigations of this sort are but a very special phase of modern analysis, and that even the specialist in the field of analysis may never need to trouble himself about the integrals of other functions than those which are continuous except at a finite number of points. That which is essential for every mathematician to know who has occasion to use the calculus to any extent is a simple formulation of the theorems and simple tests for the validity of the processes of the calculus which have been handed down to us from Euler's time and earlier:—when may a convergent series of continuous functions be integrated term by term, when may a definite integral whose integrand satisfies reasonable conditions of continuity be differentiated under the sign of integration? These are questions of general interest to mathematicians. To the importance of a simple and lucid answer French mathematicians are alive. With full appreciation of modern standards of rigor they do not allow themselves to obscure in their presentation of the calculus the main facts of analysis by cumbersome details.

The book before us belongs to the best type of modern French treatises on the calculus. It is based on Professor Goursat's university lectures. According to the plan of instruction in France the student of mathematics learns at the *lycée* the meaning and use of derivatives, differentials not being introduced, and the rudiments of algebraic analysis. Thus the university teacher can assume that the student is familiar with the notion of the limit and the elementary methods of the calculus, and that he has sufficient maturity to understand a treatment of the calculus such as is given in American universities in a second and third course.

. . .

A treatise on advanced calculus which should present the whole subject rigorously and attractively, and at the same time in the spirit of modern analysis, has been sorely needed by students of mathematics who intend to proceed to the study of mathematical physics or of some of the various branches of analysis—theory of functions, differential equations, calculus of variations, etc. Professor Goursat's

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work meets the needs of such students in a thoroughly satisfactory manner, and we recommend it to them most heartily. The teacher of calculus will find many suggestions in the book which will enable him to improve his course, and he may often with advantage refer even an elementary class to the more elementary parts of the book for collateral reading. The range of the book is wide. While beginning with the elements of the calculus, it carries the reader to the point where he is prepared to use original sources and extracts from $\epsilon$-proofs the underlying thought. When the future historian inquires how the calculus appeared to the mathematicians of the close of the nineteenth century, he may safely take Professor Goursat's book as an exponent of that which is central in the calculus conceptions and methods of this age.

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February, 1903