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Einfachste Grundbegriffe der Topologie, by Paul Alexandroff, with an introduction by David Hilbert, Julius Springer, Berlin, 1932, 48 pp.

Einführung in die kombinatorische Topologie, by Kurt Reidemeister, Vieweg, Braunschweig, 1932, xii + 209 pp.

Knotentheorie (vol. 1 of the Ergebnisse der Mathematik und ihrer Grenzgebiete), by K. Reidemeister, Julius Springer, Berlin, 1932, vi + 114 pp. and 114 figures

To the uninitiate in matters of combinatorial analysis situs the first two of these books on topology will seem hardly to deal with the same subject, so completely do they complement each other. The little book of Alexandroff is an admirable and simple exposition of the ideas centering around homology and dimension, while Reidemeister's book, more complete and more important scientifically, is concerned with the study of abstract discrete groups and their topological applications.

Alexandroff has written particularly for those who do not care to undertake a systematic study of topology, and his enthusiasm in exhibiting the beauties of the subject will not escape the reader. There are many simple examples and the accompanying drawings are so skillfully made that one can not fail to see how the homology group operates, or to appreciate its intuitive meaning. The author does not allow the proofs of important matters to depend on the reader's intuition; on the contrary, it is remarkable how much he has treated in complete scientific detail. There is for example a very readable proof of the invariance of the Betti numbers (modeled after an elegant proof by Alexander) and a proof of the author's own important deformation theorem, with a corollary in the theory of dimensionality. The reader who wishes to learn more about these subjects will know where to begin from the numerous references to other authors; but for those who do not intend to consult the original papers, the references will not always give a complete nor accurate impression of the history of the subject. For example, the first formulation of the very important "Pflastersatz" was by Lebesgue in 1911, a fact that might well have been mentioned, despite the incompleteness of Lebesgue's proof. And again, what the author presumably means by the Lefschetz-Hopf fixed-point formula is merely due to Lefschetz, and should be named, we believe, accordingly.

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In recent years the works of Dehn, Nielsen, Reidemeister, Schreier, and others have brought about something like the beginnings of a general theory of discrete groups, and it is precisely to this theory that Reidemeister's *Einführung* is devoted.

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In his *Knotentheorie* Reidemeister is less severe with the reader, and various types of knot diagrams and operations on them are nicely illustrated. There are a number of ways of associating a matrix or a group with a knot, and the problem of classifying knots then becomes one of finding such calculable invariants of the groups and matrices as are also *knot invariants*. Practically all known results along these lines, including several hitherto unpublished, are included in this work, and

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the author's own contributions in which the interrelations of the various groups are developed serve effectively to unite the existing theory. The knot problem is perhaps the oldest problem in analysis situs. Gauss defined looping coefficients, and Listing, in the first published *Topologie* of mathematics, devoted much space to knots. Later, the knot problem assumed a temporary importance in physics. An atom was thought of as a knotted vortex in the ether and it was natural to suppose that the properties of an element depended on the knot structure of its atoms. More recently, it has been pointed out that the problem of classifying three-dimensional manifolds may be intimately connected with that of classifying knots. In one of Tait's classic papers on knots (1879), he admits that he "has not succeeded in catching the right note." Today we have a "theory" and have built much ingenious mathematics around the concept of knot. But what more do we actually know about knots themselves? The answer seems to be this: any knot whose plane projection contains nine or fewer double points can be recognized as belonging to one of a finite number of non-equivalent types. This meager knowledge depends partly on the empirical results of Tait and his contemporaries, partly on the known calculable knot invariants such as those discovered by Alexander, and partly on special results (some of which are still unpublished) concerning pairs of knots which are not distinguishable by their invariants. Thus the problem is still open and still fascinating—the more so since it is now apparent that even though the problem seems to be one of abstract groups, progress may depend on the results of the most unexpected domains of algebra. The complete and concise little work of Reidemeister will do much to encourage further attacks.

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