

Homological algebra, by Henri Cartan and Samuel Eilenberg, The Princeton University Press, Princeton, 1956, 15 + 390 pp., \$7.50

At last this vigorous and influential book is at hand. It took nearly three years from completed manuscript to bound book; Princeton is penalized 15 yards for holding.

Homological algebra deals both with the homology of algebraic systems and with the algebraic aspects of homology theory. The first topic includes the homology and cohomology theories of groups, of associative algebras, and of Lie algebras. The second topic includes the care and feeding of exact sequences and spectral sequences, as well as the manipulation of functors of chain complexes. For example, the Künneth problem reads: Given the homology of complexes K and L , what is the homology of $K \otimes L$? Again, the universal coefficient problem reads: Given a group G and the homology of a complex K , what is the homology of the complexes $K \otimes G$ and $\text{Hom}(K, G)$? These problems and these two functors, tensor product and Hom , are treated not just for groups, but in proper generality for left modules over an arbitrary ring Λ .

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Perhaps Mathematics now moves so fast—and in part because of vigorous unifying contributions such as that of this book—that no unification of Mathematics can be up to date. The reviewer might also add his strictly personal opinion that the authors have not kept sufficiently in mind the distinction between a research paper and a book: a good research paper presents a promising new idea when it is hot—and when nobody knows for sure that it will turn out to be really useful; a good research book presents ideas (still warm) after their utility has been established in the hands of several workers.

This book contains too large a proportion of shiny new ideas which have nothing to recommend them but their heat and promise: satellites (these appear in Chapter III and then gradually disappear in later chapters), derived functors of anything but Hom and \otimes (the reviewer watched in vain for other examples), semi-hereditary rings, functors derived simultaneously in several variables, supplemented algebras, and the homology of monoids. The same remark applies to spectral sequences. These sequences *have* proved their worth in topology but have not yet reached decisive results in the homology of algebraic systems: the result is that the uninitiated reader can hardly hope to understand what spectral sequences are all about by reading the three chapters devoted to them in this book. The reviewer is not claiming that spectral sequences and these other notions will not later have significant algebraic uses: some of them will, but until that time comes their presence clutters up the book.

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S. MAC LANE