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Éléments de géométrie algébrique, by A. Grothendieck, rédigés avec la collaboration de J. Dieudonné, Publications de l'Institut des Hautes Études Scientifiques No. 4, Paris, 1960, 228 pp., 27 NF

The present work, of which Chapters 0 and 1 are now appearing together, is one of the major landmarks in the development of algebraic geometry. It plans to cover eventually everything that is known in algebraic geometry over arbitrary ground rings, and of course a lot more besides. A tentative list of its chapters is as follows:

. . .

In order to get a more specific idea of what is to come, one should consult first Grothendieck's address to the International Congress at Edinburgh, 1958, and also the whole series of talks at Bourbaki seminars given in the past two years (available at the Institut Henri Poincaré, 11 Rue Pierre Curie, Paris) in which he has given a sketch of the proofs of important results to appear in later chapters. These talks will provide the necessary motivation to the whole work. They are written concisely, directly, and excitingly. Such motivation could not be given in the actual text, which is written very lucidly, is perfectly organized, and very precise. Thanks are due here to Dieudonné, without whose collaboration the labor involved in writing and publishing the work would have been insurmountable.

. . .

In algebraic geometry, Grothendieck reformulates certain classical problems in terms of the representation of functors, for instance the problem of constructing Picard schemes. Given X over S, the Picard functor consists in associating to each T over S the divisor classes of X which are rational over T. (This can of course be made precise.) The Picard scheme, if it exists, represents this functor. Grothendieck has recently obtained a fairly general condition on functors in the category of schemes under which he can prove that a functor is representable. This point of view marks a complete discontinuity with those preceding it and in a certain sense, is the first essentially new approach having entered algebraic geometry since the Italian school.

A theorem is not true any more because one can draw a picture, it is true because it is functorial.

To conclude this review, I must make a remark intended to emphasize a point which might otherwise lead to misunderstanding. Some may ask: If Algebraic Geometry really consists of (at least) 13 Chapters, 2,000 pages, all of commutative algebra, then why not just give up?

The answer is obvious. On the one hand, to deal with special topics which may be of particular interest only portions of the whole work are necessary, and shortcuts can be taken to arrive faster to specific goals. Thus one may expect a period of coexistence between Weil's *Foundations* and *Elements*. Only history will tell if one buries the other. Projective methods, which have for some geometers a particular attraction of their own, and which are of primary importance in some

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aspects of geometry, for instance the theory of heights, are of necessity relegated to the background in the local viewpoint of *Elements*, but again may be taken as starting point given a prejudicial approach to certain questions.

But even more important, theorems and conjectures still get discovered and tested on special examples, for instance elliptic curves or cubic forms over the rational numbers. And to handle these, the mathematician needs no great machinery, just elbow grease and imagination to uncover their secrets. Thus as in the past, there is enough stuff lying around to fit everyone's taste. Those whose taste allows them to swallow the *Elements*, however, will be richly rewarded.

S. Lang