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ABOUT THE COVER: JOHN WALLIS AND OXFORD

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John Wallis (1616–1703) was very likely the most versatile and influential English mathematician prior to the appearance of Isaac Newton. But if one mentions his name today most current mathematicians would have trouble coming up with any of his accomplishments other than a formula from calculus, the well-known Wallis product,

$$\pi/2 = \frac{2 \times 2 \times 4 \times 4 \times 6 \times 6 \cdots}{1 \times 3 \times 3 \times 5 \times 5 \times 7 \cdots},$$

a formula for approximating the value of π. Carl Boyer in [1, pp. 420–421] flatly states that this was first discovered by Wallis, thus running counter to Boyer's Law that states that formulas and theorems are usually not named for their initial discoverers. Here is an exception. (Boyer's Law is, however, a good example of Boyer's Law, since it was certainly observed earlier and "is a rare instance of a law whose statement confirms its own validity!" [4].) Apparently Wallis discovered the product while looking at what we now know as the integral to calculate the area of a circle. He did this prior to Newton's first publication on the calculus and hence was carrying out calculations that were precursors of calculus as early as 1655. Much of this work appeared in his *Arithmetica infinitorum* of that year.

A short list of Wallis's mathematical contributions follows: the introduction of the lemniscate-like symbol for infinity; the extension of the use of exponents to include negative and rational numbers; a study of various infinite series well before they were adequately defined; the introduction of the term "continued fraction" and the work of his student, William, Viscount Brouncker, in transforming Wallis's product into a continued fraction [2]. Also in 1655 Wallis published a pioneering treatise on conic sections that survived as a textbook at Cambridge for years. Later in life, in 1685, he wrote *A Treatise of Algebra*. The portrait of Wallis (Figure 1) is opposite the title page in that volume. The book appeared in both English and Latin (*De algebra tractatus*) [7, p. 88]. In this we find Wallis anticipating the topic of Gauss's doctoral dissertation, the Fundamental Theorem of Algebra, and

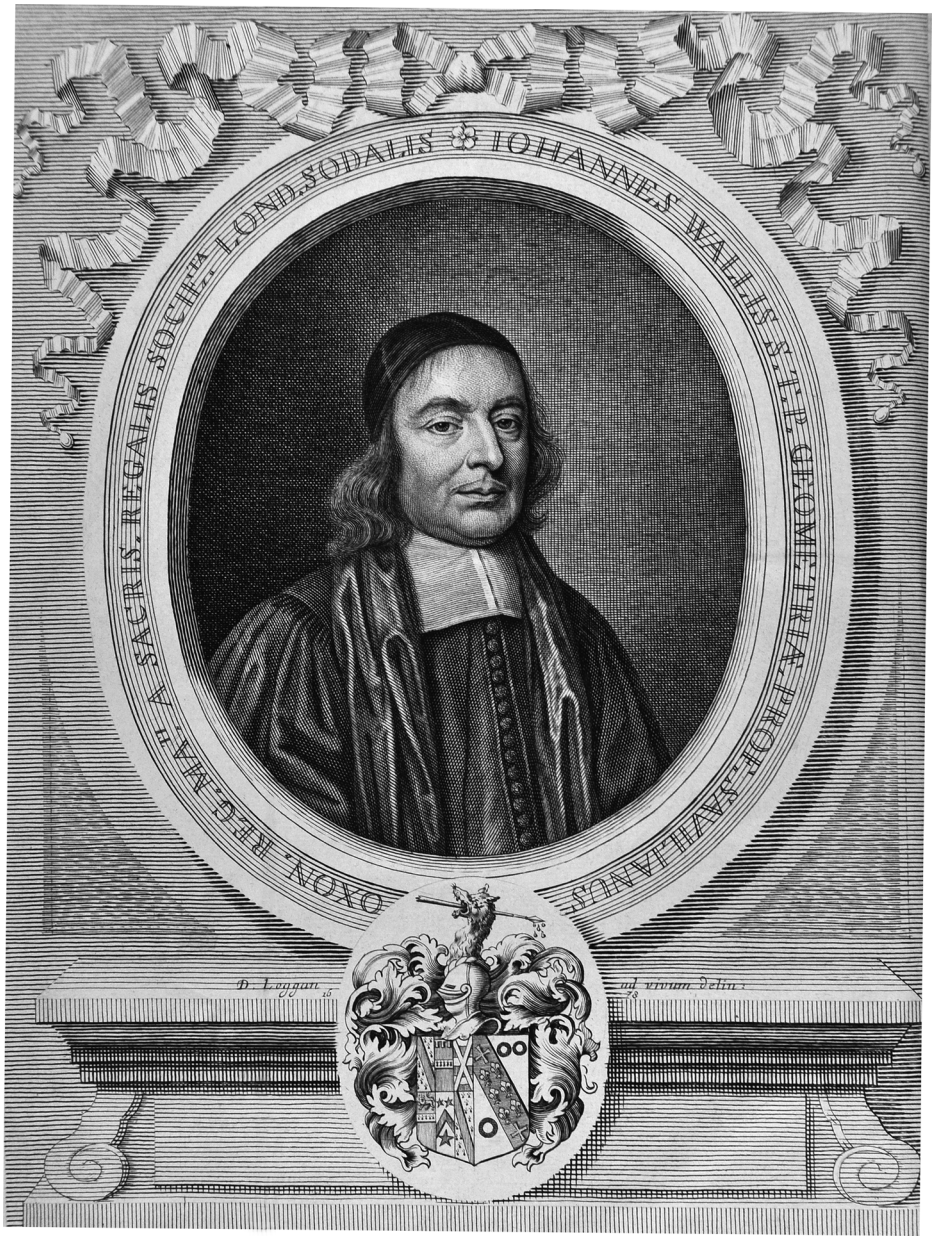

FIGURE 1. Portrait of John Wallis from *A Treatise of Algebra*.

otherwise studying the nature of roots of polynomial equations. And though he was unfamiliar with the binomial theorem, he managed to use it in special cases even though it was only later that Newton formulated it for powers of binomials other than positive integers. Newton had been attracted to the problem of binomials, one of his best-known achievements, by Wallis's *Arithmetica infinitorum*. Wallis also responded to a challenge by Pascal to solve some problems about the cycloid. This is but a sampling of mathematical problems Wallis attacked and solved. There are more, but they tend to be somewhat technical questions of rectifying curves, a task made more challenging by his lack of knowledge of the simpler techniques provided by the calculus.

Wallis's interests, however, were not restricted to mathematics. Before going to Cambridge he became proficient in Greek, Latin, and Hebrew, so that when he went to Emmanuel College, Cambridge, in 1636, he could study philosophy (ethics and metaphysics), medicine, astronomy, and geography. In 1640 he was ordained by the Bishop of Westminster and became a chaplain in various locations in England. Shortly after that he became interested in cryptography and applied his skills to decoding messages for Cromwell's intelligence service during the English Civil War. It was this interest that gave him access to a group that established the Royal Society of London in 1660. His student Brouncker was its first president.

In 1649 he was appointed Savilian Professor of Geometry at Oxford and held that position for a record 54 years, a tenure yet to be surpassed. This chair has been held by many eminent mathematicians, such as Henry Briggs (of common logarithms); Edmond Halley, the astronomer; J. J. Sylvester; G. H. Hardy; Edward Titchmarsh; Michael Atiyah; Ioan James; and Richard Taylor. Earlier he had received a fellowship at Queens' College, Cambridge, but he had to resign from that position when he married.

As a faculty member Wallis was concerned with what students were learning, so he argued for the inclusion of logic in the curriculum—not surprising since he had written a textbook on the subject in 1687. His teaching logic "was to lay 'the foundations of that learning, which they are to exercise and improve all their life after', explaining its merits as being to manage our reason to the best advantage, with strength of argument and in good order, and to apprehend distinctly the strength or weakness of another's discourse, and discover the fallacies or disorder whereby some other may endeavour to impose upon us, by plausible but empty words, instead of cogent arguments and strength of reason" [3, pp. 5–6].

Academic discourse in Oxford and Cambridge was rather more robust in Wallis's time than it is today. Thomas Hobbes, who was a philosopher but probably not much of a mathematician, nevertheless did not hesitate to write on mathematics and to comment on the mathematical work of others. He described Wallis's most important book, the *Arithmetica infinitorum*, as a "scab of symbols". Hobbes had dared to publish a book on mathematics and claimed to have "squared the circle", which even then earned the scorn of the mathematical community. (Later he claimed to have solved the classical problem of the duplication of the cube.) His book contained many mistakes and when this was pointed out, Hobbes replied to Wallis and Seth Ward, Oxford's Savilian Professor of Astronomy at the time, "So go your ways you uncivil ecclesiastics, inhuman divines, dedoctors of morality, unasinous colleagues, egregious pair of Isaachers, most wretched Vindices and Indices Academiarum. . . " [5, p. 343]. Hobbes seems clearly to have been vexed. Of course, Wallis was not

entirely a victim in these disputes. The so-called Hobbes–Wallis dispute went on for roughly 20 years, but they were probably in no hurry to end it. Wallis lived to be 87 and Hobbes died in his 90th year. Wallis no doubt antagonized Hobbes by not reacting to Hobbes's attacks and treating his arguments with disdain. Their relationship was predetermined to be a rocky one. Hobbes viewed mathematics as being primarily geometrical, indeed in a sense part of physics, whereas Wallis viewed it as arithmetical and algebraic. They could reconcile their differences only occasionally. The description of Wallis's arithmetic-algebraic tastes is only partly true: he did publish on conic sections, and the Savilian Professor of Astronomy (and mathematician) David Gregory did refer publicly to Wallis as "the prince of geometers" [3, p. 27].

The astronomer John Flamsteed was not pleased with the choice of successor to Wallis as the Savilian Professor of Geometry, Edmond Halley. In a "letter of Flamsteed's of December 1703, [he] reveals his irritation at the turn of events: Dr. Wallis is dead—Mr. Halley expects his place—who now talks, swears and drinks brandy like a sea-captain" [3, p. 128].

In another dispute, Wallis in his *A Treatise of Algebra*, "claimed that Descartes borrowed heavily from Harriot: 'Hariot hath laid the foundations on which Des Cartes (though without naming him) hath built the greatest part (if not the whole) of his Algebra on Geometry.' Wallis missed the point, but his statement foreshadows the tension between English and Continental mathematicians typified in the later Newton–Leibniz priority struggle" [6, p. 90].

John Wallis will not be forgotten in mathematical circles for a long time, nor forgotten by the literate public. He was one of the two principal characters in Iain Pears' historical mystery novel, *An Instance of the Fingerpost* (Jonathan Cape, 1997) where other members of the cast were Robert Boyle, John Locke, and Christopher Wren. The setting was a lively time in English intellectual history—the period before and after the days of Kings Charles I and II and Oliver Cromwell, mentioned earlier.

In 1969 the Oxford Mathematical Institute set up the Wallis Professorship, currently held by Terry Lyons and earlier held by Simon Donaldson, one of the Fields Medalists named at the International Congress of Mathematicians in Berkeley in 1986. So the Wallis name lives on.

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