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ABOUT THE COVER: VIGELAND'S ABEL MONUMENT

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The featured article in this issue by John Milnor is an extended version of his Abel Lecture at the 2014 International Congress of Mathematicians. Inspired by the connection, we use the famous memorial to Abel as the cover of this issue of the *Bulletin* and discuss a few aspects of Abel's life.

Great monuments made of bronze and stone honoring mathematicians are rare. But there is one definitely worth noting: the dramatic and memorable statue (1908) of Niels Henrik Abel (1802–1829) by Gustav Vigeland in the Royal Park (Slottsparken) in Oslo (see Figure 1). If the frequency of being photographed by tourists is a measure of quality, then its only rival is, perhaps, the tomb and monument honoring Newton at the choir screen of Westminster Abbey. But few people visiting the Abbey go there only to view this gigantic sarcophagus with a recumbent Newton, his arm on a stack of his most important books, accompanied by a couple of putti holding scientific instruments and a slate and globe with a representation of the signs of the Zodiac (!). Above is a large globe and a seated female figure representing Astronomy. The much larger statue of Abel is, by contrast, less conventional and a favorite destination for many. Of course, crowds in Oslo do not match those in Westminster Abbey.

There are other popular mathematical tombs, of course, such as Euler's heavy and somber tomb at the Alexander Nevsky Monastery in St. Petersburg, but it is no match for Vigeland's exuberant monument to Abel, which shows the mathematician as a heroic figure, first seen by many pilgrims who approach it from downtown Oslo by climbing a hill and suddenly being faced with a 12-foot high bronze statue of Abel on a 24-foot plinth of rough-hewn granite. The bronze figure with windblown hair stands astride two vanquished prone figures that ostensibly represent the genii of thought and spirit [6, p. 74]. Gábor Szegő, quoting Felix Klein, on

FIGURE 1. CLEAR COPY OF THE COVER. Vigeland's Abel Monument

FIGURE 2. Vigeland's Abel Monument (detail)

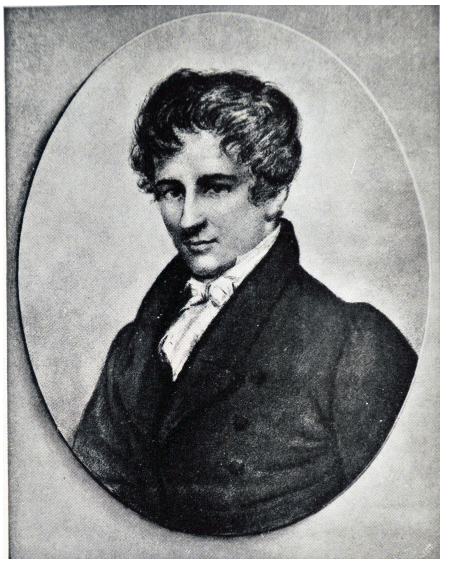


FIGURE 3. Gøbitz's portrait of Abel, 1826

the other hand, claimed that the two figures represented, respectively, the fifth-degree polynomial equation and an elliptic function, thought by some to be Abel's two greatest mathematical achievements. Distinguishing the two prone figures is a challenge, so it is not clear which one is the better likeness to the fifth-degree polynomial equation. Indeed, we cannot even be sure that Abel's face is a good likeness (see Figure 2).

Vigeland was the most capable and admired sculptor in Norway of that era. He, like other sculptors and painters, had to rely on the only portrait of Abel that is known to be authentic, a painting of 1826 by Johan Gøbitz (see Figure 3). There are, of course, other statues of the subject: a seated Abel by Gustav Laerum, to recognize the centennial of Abel's birth in 1902 and now at Arendal; another seated Abel by Ingebrigt Vik for the same occasion; a bas relief by Svein Magnus Håvarstein of 1979 at the Finnøy Vicarage; and the bust shown in the Milnor article in this issue, the work of Brynjulf Bergslien that one finds in the garden of the Gjerstad Vicarage and commissioned by Oystein Øre in 1958. We see near the base of the plinth a lemniscate, a curve much studied by Abel and appearing in some of his surviving manuscripts. If one needs reminding, it should be pointed out that Abel was born in Finnøy but grew up in Gjerstad [4]. Another casting of this very attractive bust can be found in the mathematics library at Blindern at the University of Oslo. Of course, students of Abel culture will recall that this same portrait, common to so many likenesses of Abel, also appeared on four Norwegian postage stamps (in 1929, the one hundredth anniversary of Abel's death) and on two 500 kroner banknotes (1948–1991) of the Bank of Norway [7, pp. 503, 510–511]. Here we should make clear that none of these monuments appears at the grave of Abel which is in Froland, Norway, where he was visiting his fiancé at the time of his death. The obelisk tombstone, though handsome, is not remarkable. So which

likeness is correct—Gøbitz's pensive, sensitive, perhaps even shy young man, or Vigeland's heroic and assertive one? Klein claimed it was the former.

Abel is one of the great romantic figures of mathematics and, in that context, is often paired with Évariste Galois. Both achieved fame by working successfully on a then-major problem, the proof of the insolvability in general of the fifth-degree polynomial equation by radicals. It settled a question that prompted work by masters from Descartes to Lagrange. And it was one of the problems of that period that had profound consequences on the way people looked at mathematics. The proof showed that sometimes no amount of effort expended can lead to a solution of a problem. Preceded a few years earlier by work of Lagrange and Ruffini, its solution by Abel (when it finally became known throughout Europe) was a surprising and deep result that pointed the way to much beautiful mathematics. Galois provided, in some ways, an even more productive path. Because of their contributions to this famous problem and their both dying young—Abel at age 26, Galois at 21—their lives in the minds of some were mathematical analogues of those of great English poets like Keats and Shelley—well, in one case maybe a better analogue is Lord Byron. The causes of death were equally romantic—consumption (tuberculosis) in one case and being shot in a duel in the other.

Galois's work, since he died at an even earlier age than Abel, was indeed spare, his *Œuvres mathématiques* totaling in 8vo format only 64 pages [3]. And that included even a letter or two. Abel's collected papers, on the other hand, appeared (in the revised edition of 1881) in 962 folio pages—neither comparable, to be sure, to Euler's 75 to 80 folio volumes in his *Opera omnia*, nor to Cauchy's massive collected works—but still substantial. Gathering together Abel's work was somewhat complicated, however, and on this something needs to be said.

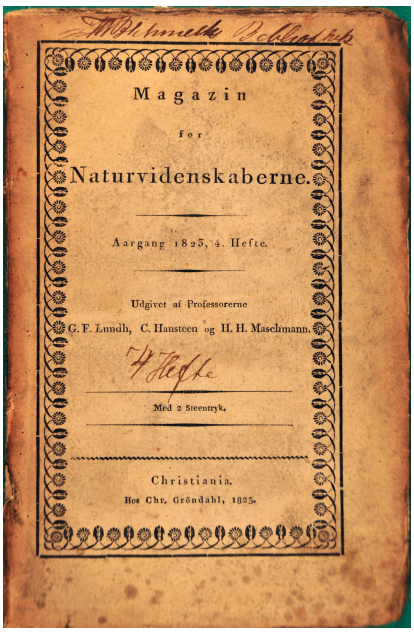
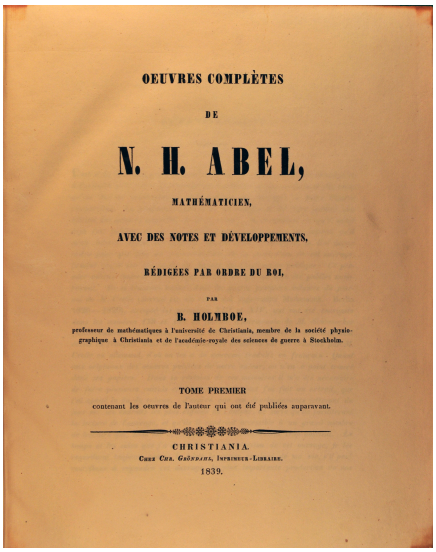

Magazin
for
Naturvidenskaberne.
Aargang 1823, 4. Hefte.
Udgivet af Professorerne
G. F. Lundh, C. Hansteen og H. H. Maschmann.
Med 2 Steentryk.
Christiania.
Hos Chr. Gröndahl, 1823.

FIGURE 4. Cover of *Magazin for Naturvidenskaberne,* 1823, 4. Hefte

As anyone interested in Abel knows, he suffered early on from being well apart from the scientific centers of Europe in his day—Paris, Berlin, and Göttingen. Few people in Norway were even qualified to recognize his genius, and from Norway it was difficult to communicate with the people in mathematics who really mattered. He began to publish his discoveries in a small Norwegian science journal, the *Magazin for Naturvidenskaberne* that was published by Gröndahl between 1823 and 1836 (see Figure 4). This had few readers outside Norway. The author of this note was lucky enough to acquire by accident a run of this journal from a small used-book store in Oslo in 1970 and it is interesting to see in some cases the evolution of some of these pieces, written when Abel was a student and originally published in Norwegian. In those years he made an occasional mathematical error. Then later when he sent more mature manuscripts off to people who could have understood them and recognized his genius, they were sometimes cast aside as the work of a crank (Gauss), or misplaced for years (Cauchy), or put aside unread till after Abel's death (the Académie des Sciences, Paris.) From time to time, though, he did have some luck. The best evidence of this was his getting to know August Crelle who, at that critical time in mathematical history, established in 1826 the *Journal für die reine und angewandte Mathematik*, often called, for good reason, "Crelle's Journal". In this journal, during his last years, Abel published some of his best work.

Abel's longtime Norwegian mentor, Bernt Holmboe, in 1839 edited and published Abel's collected papers [1], a significant collection of the work known at the time. Published in Oslo, it did not have the wide dissemination it deserved and quickly became a book that was hard to find. Further it contained some very early work that was of questionable value, one such having been described by the editors of the 1881 version of the collected works thus: "*il s'était glissé, par inadvertance, une faute grave*" [2, p. iii]. Then too, manuscripts that had been lost had in some cases resurfaced so they could be included in a later edition. There was pressure from


OEUVRES COMPLÈTES

DE

N. H. ABEL,

MATHÉMATICIEN,

AVEC DES NOTES ET DÉVELOPPEMENTS,

RÉDIGÉES PAR ORDRE DU ROI,

PAR

B. HOLMBOE,

professeur de mathématiques à l'université de Christiania, membre de la société physiographique à Christiania et de l'académie-royale des sciences de guerre à Stockholm.

TOME PREMIER

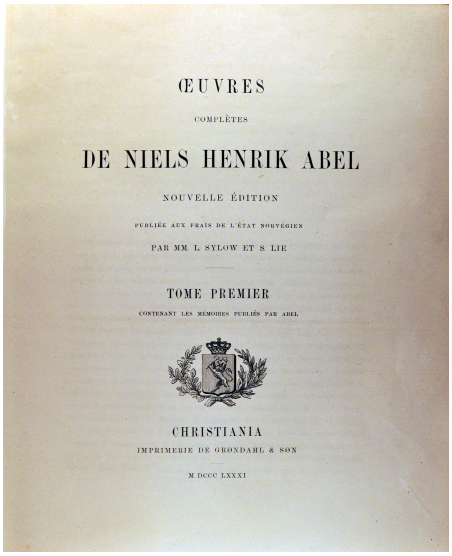
contenant les oeuvres de l'auteur qui ont été publiées auparavant.

CHRISTIANIA.

Chez Chr. Gædickens, Imprimeur-Libraire.

1839.

FIGURE 5. Title page of Abel's collected papers, 1839


ŒUVRES

COMPLÈTES

DE NIELS HENRIK ABEL

NOUVELLE ÉDITION

PUBLIÉE AUX FRAIS DE L'ÉTAT NORVÉGIEN

PAR MM. L. SYLOW ET S. LIE

TOME PREMIER

CONTENANT LES MÉMOIRES PUBLIÉS PAR ABEL

CHRISTIANIA

IMPRIMERIE DE GRONDAHL & SON

M DCCC LXXXI

FIGURE 6. Title page of Abel's collected papers, 1881

mathematicians abroad to have a new set of collected papers issued that would not only avoid some flawed work but include papers previously unavailable. So in 1881 a new and considerably improved set of Abel's papers was published again by Gröndahl, but this time edited by two internationally recognized Norwegian mathematicians, Sophus Lie and Ludwig Sylow.

Tracking Abel's papers becomes more than usually challenging when a piece started in the *Magazin for Naturvidenskaberne* as, for example, "Oplösning af nogle Opgaver ved Hjelp af bestemte Integraler, af Student N. H. Abel", Aargang 1823, 4. Hefte, pp. 205–215, is then transformed into "Solution de quelques problèmes à l'aide d'intégrales définies" in the 1881 edition (see Figure 6), with yet another name,"Résolution de quelques problèmes à l'aide d'intégrales définies," in the 1839 volume (see Figure 5). On the plus side, it prompts one to brush up on one's mathematical Norwegian.

It is wise to keep some of Abel's papers from falling into the hands of innocent children. Many of his papers are a sea of integrals, and it is not uncommon to see improper definite integrals with an upper limit of $\frac{1}{0}$, not the usual ∞.

The 1881 edition of the collected papers contains a memoir by Abel, "Sur la résolution algébrique des équations" that includes a passage that puts Ruffini in his place: "*Le premier, et, si je ne me trompe, le seul qui avant moi ait cherché à démontrer l'impossibilité de la résolution algébrique des équations générales, est le géomètre Ruffini; mais son mémoire est tellement compliqué qu'il est très difficile de juger de la justesse de son raisonnement. Il me paraît que son raisonnement n'est pas toujours satisfaisant. Je crois que la démonstration que j'ai donnée dans le premier cahier de ce journal, ne laisse rien à désirer du côté de la rigueur; mais elle n'a pas toute la simplicité dont elle est susceptible. Je suis parvenu à une autre démonstration, fondée sur les mêmes principes, mais plus simple, en cherchant à résoudre un problème plus général.*" Abel says it gently and kindly: Ruffini's proof [5] is long and complicated, but, what's worse, it's wrong!

References

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[5] Paolo Ruffini, *Teoria generale delle equazioni in cui si dimostra impossibile la soluzione algebraica delle equazioni generali di grado superiore al quarto di Paolo Ruffini*, Stamperia di S. Tommaso d'Aquino, Bologna, 1799.

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